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A Meshfree Radial Point Interpolation Method for Free Vibration of Laminated Composite Plates Analysis Based on Layerwise Theory

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Abstract

A meshfree radial point interpolation method for free vibration analysis of laminated composite plates is presented in this paper. The idea approach bases on integrating a layerwise theory with the RPIM shape function constructed by the modified Gaussian function. In the layerwise theory, proposed by Ferreira A.J.M. [1] in 2005, the displacement field is assumed first-order shear deformation theory (FSDT) with independent rotations in each layer, and imposed displacement continuity at contact position of the layers interfaces. Hereby, that easily improves the accuracy of transverse shear stresses and the using of shear correction factors is not required. The high performances of present method are considered by investigating characteristic properties of laminated composite through numerical examples. The obtained results are discussed and compared with other previous solutions in the literature.

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Keywords: Radial point interpolation method; laminated composite plate; layerwise theory

1. Introduction

Composite materials play a key role not only in scientific disciplines but also in practice due to the development of material technology. They are being applied in variety engineering, such as aerospace, automotive, marine, civil engineering, etc. Made by putting several orthotropic layers with different materials together, laminated composite materials achieved excellent engineering characteristics than the typical materials, for examples, high-stiffness, high-

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strength, lightweight, long fatigue life, corrosion resistance, thermal properties [2]. In order to use them effectively, having a perceptive understanding about their behavior subjected to loading is clearly needed.

Layerwise plate theory is successfully developed using multiquadrics by Ferreira *et al.* [1] and then is recently extended using isogeometric analysis (IGA) by Thai *et al.* [3]. Herein, this paper mentions the application of RPIM on studying free vibration analysis of laminated composite plate structures. This paper is arranged as following. The second section is the introduction of RPIM; the equations of system using RPIM based on layerwise theory are formulated for laminated composite plates in Section 3. Two numerical examples are considered in Section 4 and compared with other published solutions. In the end, the conclusions are given and discussed in Section 5.

2. Radial Point Interpolation Method (RPIM)

Let us consider a support domain $\Omega_{\mathbf{x}} \in \Omega$ corresponding to node $\mathbf{x}$ that has a set of arbitrarily distributed nodes $P_i(\mathbf{x})$ (i = 1, 2, …, n), as shown in Fig. 1.

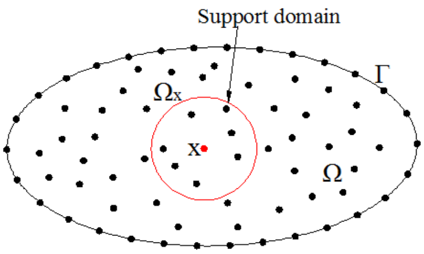

Fig. 1. Domain representation and support domain of 2D model.

The approximation function $u(\mathbf{x})$ can be estimated to all values of nodes within the support domain based on Radial Point Interpolation Method (RPIM) by combining radial basis function $B_i(\mathbf{x})$ and polynomial basis function $p_j(\mathbf{x})$. Nodal value of approximation function evaluated at the node $\mathbf{x}_i$ inside support domain is assumed to be u_i.

$$u(\mathbf{x}) = \sum_{i=1}^{n} B_i(\mathbf{x}) a_i + \sum_{j=1}^{m} p_j(\mathbf{x}) b_j = \mathbf{B}^T(\mathbf{x})\mathbf{a} + \mathbf{p}^T(\mathbf{x})\mathbf{b} \tag{1}$$

where a_i, b_j are the coefficient for $B_i(\mathbf{x})$ and $p_j(\mathbf{x})$, respectively. n is the number of scatter nodes in support domain, m is the number of polynomial basis (usually, $m < n$). The terms in Eq. (1) are defined as following

$$\begin{aligned} \mathbf{a}^T &= [a_1, a_2, a_3, \ldots, a_n] \\ \mathbf{b}^T &= [b_1, b_2, b_3, \ldots, b_m] \end{aligned}; \qquad \begin{aligned} \mathbf{a}^T &= [a_1, a_2, a_3, \ldots, a_n] \\ \mathbf{b}^T &= [b_1, b_2, b_3, \ldots, b_m] \end{aligned} \tag{2}$$

Radial basis function $B_i(\mathbf{x}) = B_i(r_i) = B_i(x, y)$ is a function of distance r_i that has the following general form, in which $r_i = \|\mathbf{x} - \mathbf{x}_i\|$ is a distance between interpolating point (x, y) and the node (x_i, y_i).

In the case of two-dimensional problem, polynomial function chosen from Pascal's triangle is expressed as

$$\mathbf{p}^T(\mathbf{x}) = [1, x, y, x^2, xy, y^2, \ldots] \tag{3}$$

To determine the coefficients a_i and b_j, the interpolation is enforced pass through all n scattered nodal points within the support domain. The interpolation at the k^{th} point has the form

$$u_k = u(x_k, y_k) = \sum_{i=1}^{n} a_i B_i(x_k, y_k) + \sum_{j=1}^{m} b_j p_j(x_k, y_k);\ k = 1, 2, \ldots, n \tag{4}$$

The following imposed constraints are used to guarantee unique approximation of a function that is the polynomial term has to satisfy an extra requirement [4]

$$\sum_{i=1}^{n} p_j\left(x_i, y_i\right) a_i = 0 \; ; \; j = 1, 2, ..., m \tag{5}$$

Eq. (5) can be written in matrix form

$$\mathbf{P}_0^T \mathbf{a} = \mathbf{0} \tag{6}$$

Combining Eq. (6) and Eq. (4) gives

$$\begin{bmatrix} \mathbf{B}_0 & \mathbf{P}_0 \\ \mathbf{P}_0^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}^e \\ \mathbf{0} \end{Bmatrix} \text{ or } \mathbf{G} \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}^e \\ \mathbf{0} \end{Bmatrix} \tag{7}$$

in which the vector for function values is defined as $\mathbf{u}^e = \left[u_1, u_2, u_3, ..., u_n\right]^T$

The moment matrix $\mathbf{B}_0$ corresponding to the radial function basis on unknown **a** and the moment matrix $\mathbf{P}_0$ on unknown **b** are

$$\underset{(n\times n)}{\mathbf{B}_0} = \begin{bmatrix} B_1(x_1,y_1) & B_2(x_1,y_1) & ... & B_n(x_1,y_1) \\ B_1(x_2,y_2) & B_2(x_2,y_2) & ... & B_n(x_2,y_2) \\ ... & ... & ... & ... \\ B_1(x_n,y_n) & B_2(x_n,y_n) & ... & B_n(x_n,y_n) \end{bmatrix} ; \; \underset{(n\times m)}{\mathbf{P}_0} = \begin{bmatrix} P_1(x_1,y_1) & P_2(x_1,y_1) & ... & P_m(x_1,y_1) \\ P_1(x_2,y_2) & P_2(x_2,y_2) & ... & P_m(x_2,y_2) \\ ... & ... & ... & ... \\ P_1(x_n,y_n) & P_2(x_n,y_n) & ... & P_m(x_n,y_n) \end{bmatrix} \tag{8}$$

The distance is directionless, hence $B_k\left(x_i, y_i\right) = B_i\left(x_k, y_k\right)$, as the result, $\mathbf{B}_0$ is a symmetric matrix. Unique solution for coefficients **a** and **b** is obtained if the inverse of matrix **G** exists,

$$\begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \mathbf{G}^{-1} \begin{Bmatrix} \mathbf{u}^e \\ \mathbf{0} \end{Bmatrix} \tag{9}$$

Finally, the interpolation is expressed as

$$u(\mathbf{x}) = \left[\mathbf{B}^T(\mathbf{x}) \quad \mathbf{p}^T(\mathbf{x})\right] \mathbf{G}^{-1} \begin{Bmatrix} \mathbf{u}^e \\ \mathbf{0} \end{Bmatrix} = \boldsymbol{\varphi}(\mathbf{x}) \mathbf{u}^e \tag{10}$$

where the matrix of shape functions $\boldsymbol{\varphi}(\mathbf{x})$ is defined by

$$\boldsymbol{\varphi}(\mathbf{x}) = \left[\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), ..., \phi_i(\mathbf{x}), ..., \phi_n(\mathbf{x})\right] \text{ in which } \phi_k(\mathbf{x}) = \sum_{i=1}^{n} B_i(\mathbf{x}) \bar{G}_{i,k} + \sum_{j=1}^{m} p_j(\mathbf{x}) \bar{G}_{n+j,k} \tag{11}$$

where $\bar{G}_{i,k}$ is the term located at (*i*, *k*) position of matrix $\mathbf{G}^{-1}$. After radial basis functions are determined, shape functions depend only upon the position of scattered nodes.

It can be observed that the radial basis functions involve the correlation parameter, so as the quality of the RPIM heavily depends on choosing the value of such the correlation parameter. As a result, the radial basis functions are affected by the correlation parameter and the number of nodes in the support domain. In order to finding a unique value of the correlation parameter for various problems, a quartic spline function introduced in this paper, is similar to the weight function in moving least square interpolation [5]

$$B_i\left(x,y\right)=\exp\left[-\left(r_i/r_{\max}\right)^2\right],\qquad \left(0\le r_i/r_{\max}\le 1\right) \tag{12}$$

where $r_{\max}$ is chosen to be the maximum distance between a pair of nodes in the support domain. From Eq. (12), the radial basis function with dimensionless value does not depend on the correlation parameter, therefore, the obtained solutions are stable and accurate.

The selection of the radius of the influence domain is the other important matter which must be considered in meshfree methods. So determining the number of scattered nodes within an interpolated domain is needed to evaluate the size of support domain.

$$d_m=\beta d_c \tag{13}$$

where d_c is a characteristic length related to the nodal spacing while β stands for a scale factor. It is known that the radius of the influence domain d_m needs to be taken large enough to cover neighboring nodes certainly.

3. A meshfree RPIM formulation for laminated composite plates using layerwise theory

3.1. The displacements, strains and stresses in plates

In this paper, the layerwise displacement field [1] of general laminated composite plates is applied to analyze. This model is assumed that the first-order shear deformation theory is treated in each layer and the continuity of displacement is imposed at the layers interfaces. In the simplest case, a three-layer symmetric laminated composite plate is chosen to formulate the present model.

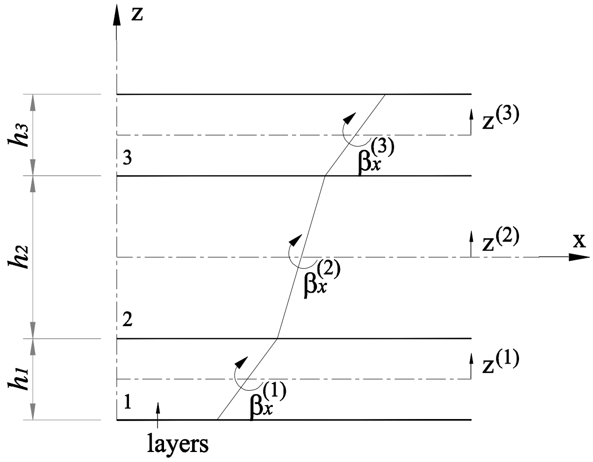

Fig. 2. 1D representation of the layerwise kinematics.

The displacement field applied for the middle layer is expressed as

$$\begin{aligned}u^{(2)}\left(x,y,z\right)&=u_0\left(x,y\right)+z^{(2)}\beta_x^{(2)}\left(x,y\right)\\ v^{(2)}\left(x,y,z\right)&=v_0\left(x,y\right)+z^{(2)}\beta_y^{(2)}\left(x,y\right)\\ w^{(2)}\left(x,y,z\right)&=w_0\left(x,y\right)\end{aligned} \tag{14}$$

$$\begin{aligned} u^{(3)}(x,y,z) &= u_0(x,y) + (h_2/2)\beta_x^{(2)} + (h_3/2)\beta_x^{(3)} + z^{(3)}\beta_x^{(3)} \\ v^{(3)}(x,y,z) &= v_0(x,y) + (h_2/2)\beta_y^{(2)} + (h_3/2)\beta_y^{(3)} + z^{(3)}\beta_y^{(3)} \\ w^{(3)}(x,y,z) &= w_0(x,y) \end{aligned} \tag{15}$$

$$\begin{aligned} u^{(1)}(x,y,z) &= u_0(x,y) - (h_2/2)\beta_x^{(2)} - (h_1/2)\beta_x^{(1)} + z^{(1)}\beta_x^{(1)} \\ v^{(1)}(x,y,z) &= v_0(x,y) - (h_2/2)\beta_y^{(2)} - (h_1/2)\beta_y^{(1)} + z^{(1)}\beta_y^{(1)} \\ w^{(1)}(x,y,z) &= w_0(x,y) \end{aligned} \tag{16}$$

where h_k and $z^{(k)} \in [-h_k/2, h_k/2]$ are the k^{th}-layer thickness and z coordinates, respectively.

The in-plane and shear strain components of layer k^{th} are given by

$$\left\{ \varepsilon_{xx}^{(k)} \quad \varepsilon_{yy}^{(k)} \quad \gamma_{xy}^{(k)} \quad \gamma_{xz}^{(k)} \quad \gamma_{yz}^{(k)} \right\} = \left\{ u_{,x}^{(k)} \quad v_{,y}^{(k)} \quad u_{,y}^{(k)} + v_{,x}^{(k)} \quad u_{,z}^{(k)} + w_{,x}^{(k)} \quad v_{,z}^{(k)} + w_{,y}^{(k)} \right\} \tag{17}$$

Neglecting $\sigma_z^{(k)}$ for each orthotropic layer, the constitutive equation of an orthotropic layer in the local coordinate system is derived from Hookes law for plane stress by $\sigma_{ij}^{(k)} = Q_{ij}^{(k)}\varepsilon_{ij}^{(k)}$ where the reduced stiffness components, $Q_{ij}^{(k)}$ are given in [2].

3.2. The governing equations in weak form

In layerwise theory, the shear-correction factors do not use as in FSDT model. Here, only symmetric laminated composite plate are considered, therefore, u_0, v_0 and related stress resultants can be dismissed.

A weak form, obtained from the dynamic form of the principle of virtual work, may be derived for the free vibration analysis of laminated composite and sandwich plates using layerwise theory

$$\int_\Omega \left(\sum_{k=1}^{3} \delta\left(\boldsymbol{\varepsilon}_p^{(k)}\right)^T \bar{\mathbf{D}}^{(k)} \boldsymbol{\varepsilon}_p^{(k)} \right) d\Omega + \int_\Omega \left(\sum_{k=1}^{3} \delta\left(\boldsymbol{\gamma}^{(k)}\right)^T \mathbf{C}^{(k)} \boldsymbol{\gamma}^{(k)} \right) d\Omega = \int_\Omega \left(\sum_{k=1}^{3} \delta\left(\tilde{\mathbf{u}}^{(k)}\right)^T \mathbf{m}^{(k)} \ddot{\tilde{\mathbf{u}}}^{(k)} \right) d\Omega \tag{18}$$

where $\boldsymbol{\varepsilon}_p^{(k)} = \left\{ \varepsilon_{xx}^{(k)} \quad \varepsilon_{yy}^{(k)} \quad \gamma_{xy}^{(k)} \right\}^T$ and $\boldsymbol{\gamma}^{(k)} = \left\{ \gamma_{xz}^{(k)} \quad \gamma_{yz}^{(k)} \right\}^T$ are in-plane strains and transverse shear strains vector corresponding to k^{th}-layer, respectively, and

$$\begin{gathered} \bar{\mathbf{D}}^{(k)} = \begin{bmatrix} \mathbf{A}^{(k)} & \mathbf{B}^{(k)} \\ \mathbf{B}^{(k)} & \mathbf{D}^{(k)} \end{bmatrix}; \\ \left(A_{ij}^{(k)}, B_{ij}^{(k)}, D_{ij}^{(k)} \right) = \int_{-h_k/2}^{h_k/2} \left(1, z, z^2\right) \bar{Q}_{ij}^{(k)} dz \quad (i,j = 1,2,6); \quad \left(C_{ij}^{(k)} \right) = \int_{-h_k/2}^{h_k/2} \bar{Q}_{ij}^{(k)} dz \quad (i,j = 4,5) \end{gathered} \tag{19}$$

and

$$\mathbf{m}^{(k)} = \begin{bmatrix} I_0^{(k)} & I_1^{(k)} \\ I_1^{(k)} & I_2^{(k)} \end{bmatrix} \;;\; \left(I_0^{(k)}, I_1^{(k)}, I_2^{(k)}\right) = \int_{-h_k/2}^{h_k/2} \rho^{(k)} \left(1, z, z^2\right) dz \tag{20}$$

From Eqs. (14)-(16), the total displacement fields can be rewritten as $\mathbf{u} = \sum_{k=1}^{3} \mathbf{u}^{(k)}$. Hence $w_0^{(1)} = w_0^{(2)} = w_0^{(3)} = w$, it can be rewritten as $\mathbf{u} = \left\{ w \quad \beta_x^{(1)} \quad \beta_y^{(1)} \quad \beta_x^{(2)} \quad \beta_y^{(2)} \quad \beta_x^{(3)} \quad \beta_y^{(3)} \right\}$.

Using RPIM basis functions, approximate variables are the transverse deflection and the rotations at all nodes within the support domain, which can be expressed as

$$\mathbf{u} = \sum_{I=1}^{n} \boldsymbol{\varphi}_I \mathbf{d}_I \tag{21}$$

where n is the nodes of support domain, $\boldsymbol{\varphi}_I$ is the rational basis functions and $\mathbf{d}_I = \left\{ w_I \quad \beta_{xI}^{(1)} \quad \beta_{yI}^{(1)} \quad \beta_{xI}^{(2)} \quad \beta_{yI}^{(2)} \quad \beta_{xI}^{(3)} \quad \beta_{yI}^{(3)} \right\}$ is the degrees of freedom of $\mathbf{u}$ associated with the node I.

The in-plane strains and shear strains are written as

$$\left\{ \boldsymbol{\varepsilon}_p \quad \boldsymbol{\gamma} \right\}^T = \sum_{I=1}^{n} \left[\mathbf{B}_I^{mb1} \quad \mathbf{B}_I^{mb2} \quad \mathbf{B}_I^{mb3} \quad \mathbf{B}_I^{b1} \quad \mathbf{B}_I^{b2} \quad \mathbf{B}_I^{b3} \quad \mathbf{B}_I^{s1} \quad \mathbf{B}_I^{s2} \quad \mathbf{B}_I^{s3} \right]^T \mathbf{d}_I = \sum_{I=1}^{n} \mathbf{B}_I \mathbf{d}_I \tag{22}$$

where

$$\mathbf{B}_I^{mb1} = \begin{bmatrix} 0 & -\left(h_1/2\right)\phi_{I,x} & 0 & -\left(h_2/2\right)\phi_{I,x} & 0 & 0 & 0 \\ 0 & 0 & -\left(h_1/2\right)\phi_{I,y} & 0 & -\left(h_2/2\right)\phi_{I,y} & 0 & 0 \\ 0 & -\left(h_1/2\right)\phi_{I,y} & -\left(h_1/2\right)\phi_{I,x} & -\left(h_2/2\right)\phi_{I,y} & -\left(h_2/2\right)\phi_{I,x} & 0 & 0 \end{bmatrix};$$

$$\mathbf{B}_I^{mb2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_I^{mb3} = \begin{bmatrix} 0 & 0 & 0 & \left(h_2/2\right)\phi_{I,x} & 0 & \left(h_3/2\right)\phi_{I,x} & 0 \\ 0 & 0 & 0 & 0 & \left(h_2/2\right)\phi_{I,y} & 0 & \left(h_3/2\right)\phi_{I,y} \\ 0 & 0 & 0 & \left(h_2/2\right)\phi_{I,y} & \left(h_2/2\right)\phi_{I,x} & \left(h_3/2\right)\phi_{I,y} & \left(h_3/2\right)\phi_{I,x} \end{bmatrix} \tag{23}$$

$$\mathbf{B}_I^{b1} = \begin{bmatrix} 0 & \phi_{I,x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{I,y} & 0 & 0 & 0 & 0 \\ 0 & \phi_{I,y} & \phi_{I,x} & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_I^{b2} = \begin{bmatrix} 0 & 0 & 0 & \phi_{I,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{I,y} & 0 & 0 \\ 0 & 0 & 0 & \phi_{I,y} & \phi_{I,x} & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_I^{b3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \phi_{I,x} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_{I,y} \\ 0 & 0 & 0 & 0 & 0 & \phi_{I,y} & \phi_{I,x} \end{bmatrix} \tag{24}$$

$$\mathbf{B}_I^{s1} = \begin{bmatrix} \phi_{I,x} & \phi_I & 0 & 0 & 0 & 0 & 0 \\ \phi_{I,y} & 0 & \phi_I & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_I^{s2} = \begin{bmatrix} \phi_{I,x} & 0 & 0 & \phi_I & 0 & 0 & 0 \\ \phi_{I,y} & 0 & 0 & 0 & \phi_I & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_I^{s3} = \begin{bmatrix} \phi_{I,x} & 0 & 0 & 0 & 0 & \phi_I & 0 \\ \phi_{I,y} & 0 & 0 & 0 & 0 & 0 & \phi_I \end{bmatrix} \tag{25}$$

For free vibration analysis, the formulation of laminated composite and sandwich plates based on the layerwise theory is obtained as

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\mathbf{d} = \mathbf{0} \tag{26}$$

where $\mathbf{K}$ is the global stiffness matrix

$$\mathbf{K}=\int_{\Omega}\left(\sum_{k=1}^{3}\left[\begin{Bmatrix}\mathbf{B}^{mb(k)}\\ \mathbf{B}^{b(k)}\end{Bmatrix}^T\begin{bmatrix}\mathbf{A}^{(k)} & \mathbf{B}^{(k)}\\ \mathbf{B}^{(k)} & \mathbf{D}^{(k)}\end{bmatrix}\begin{Bmatrix}\mathbf{B}^{mb(k)}\\ \mathbf{B}^{b(k)}\end{Bmatrix}+\left(\mathbf{B}^{s(k)}\right)^T\mathbf{C}^{(k)}\mathbf{B}^{s(k)}\right]\right)d\Omega \tag{27}$$

and $\mathbf{M}$ is the global mass matrix

$$\mathbf{M}=\int_{\Omega}\left(\sum_{k=1}^{3}\begin{Bmatrix}\mathbf{N}_1^{(k)}\\ \mathbf{N}_2^{(k)}\end{Bmatrix}^T\begin{bmatrix}I_0^{(k)} & I_1^{(k)}\\ I_1^{(k)} & I_2^{(k)}\end{bmatrix}\begin{Bmatrix}\mathbf{N}_1^{(k)}\\ \mathbf{N}_2^{(k)}\end{Bmatrix}\right)d\Omega \tag{28}$$

where

$$\mathbf{N}_1^{(1)}=\begin{bmatrix}0 & -(h_1/2)\phi_I & 0 & -(h_2/2)\phi_I & 0 & 0 & 0\\ 0 & 0 & -(h_1/2)\phi_I & 0 & -(h_2/2)\phi_I & 0 & 0\\ \phi_I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix};\quad \mathbf{N}_2^{(1)}=\begin{bmatrix}0 & \phi_I & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \phi_I & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{29}$$

$$\mathbf{N}_1^{(2)}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \phi_I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix};\quad \mathbf{N}_2^{(2)}=\begin{bmatrix}0 & 0 & 0 & \phi_I & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \phi_I & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{30}$$

$$\mathbf{N}_1^{(3)}=\begin{bmatrix}0 & 0 & 0 & (h_2/2)\phi_I & 0 & (h_3/2)\phi_I & 0\\ 0 & 0 & 0 & 0 & (h_2/2)\phi_I & 0 & (h_3/2)\phi_I\\ \phi_I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix};\quad \mathbf{N}_2^{(3)}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \phi_I & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_I\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{31}$$

in which ω is natural frequency.

4. Numerical results

Let us consider a four layer [$0^0/90^0/90^0/0^0$] simply supported laminated composite plate with the material parameters are used here: $E_1 = 40E_2$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = 0.25$, $\rho = 1$. The scale factor β is taken equal 2.4 to cover large enough nodes in the support domain for existing of all the numerical problems. Now the effects of the elastic modulus ratios E_1/E_2 and length to thickness a/h are investigated. In Table 1, the first normalized frequency obtained from the present model is listed with respect to $a/h = 5$ and various modulus ratios. The results are compared with the meshfree based on FSDT using multiquadric radial basis functions (MRBF) [6], the moving least squares differential quadrature method (DQM) [7] based on FSDT, and the exact solutions based on HSDT [8]. It can be seen that in Table 1, the first normalized frequency of present solution is a little bit higher than other methods. As a result, the non-dimensional frequency parameter become greater due to the composite plate will be harder.

The effect of length to thickness ratios is also estimated through Table 2. The present results are compared with those of the global-local higher-order theory of Matsunaga [9], local higher-order theory (LHOT) of Whu and Chen [10], GLHOT model of Zhen and Wanji [11] and HSDT of Cho et al. [12]. The obtained results are good agreement with other published literatures.

Table 1. A non-dimensional frequency parameter $\bar{\omega} = \left(\omega a^2 / h\right)\left(\rho / E_2\right)^{1/2}$ of a [$0^0/90^0/90^0/0^0$] SSSS laminated square plate (a/h = 5).

Method/Authors	E_1/E_2			
	10	20	30	40
RPIM-LW	8.3321	9.5987	10.3413	10.8465
MRBF-FSDT [6]	8.2526	9.4974	10.2308	10.7329
DQM-FSDT [7]	8.2924	9.5613	10.32	10.849
Exact-HSDT [8]	8.2982	9.5671	10.326	10.854

Table 2. A non-dimensional frequency parameter $\bar{\omega} = \left(\omega a^2 / h\right)\left(\rho / E_2\right)^{1/2}$ of a [$0^0/90^0/90^0/0^0$] SSSS laminated plate (a/b = 1 and E_1/E_2 = 40).

Method	a/h						
	4	5	10	20	25	50	100
RPIM-LW	9.3549	10.8465	15.2035	17.7200	18.1282	18.7267	18.8929
Matsunaga [9]	9.1988	10.6876	15.0721	17.6369	18.0557	18.6702	18.8352
Whu and Chen [10]	9.193	10.682	15.069	17.636	18.055	18.67	18.835
Zhen and Wanji [11]	9.2406	10.7294	15.1658	17.8035	18.2404	18.9022	19.1566
Cho et al. [12]	-	10.673	15.066	17.535	18.054	18.67	18.835

5. Conclusions

A RPIM interpolation formulation based on layerwise theory has just been proposed for free vibration analysis of laminated composite plates. The expressions of RPIM formulation are plain with C^0-continuity requirement when compared with other theories. For all test cases considering the influence of lamina properties, the obtained results showed the efficiency and accuracy. Finally, the present method based on layerwise theory has been focused and proven throughout numerical vibration problems in this paper.

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