FEA review of continuum mechanics models for size-dependent analysis of beams and plates

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Abstract

This paper presents a comprehensive review on the development of higher-order continuum models for capturing size effects in small-scale structures. The review mainly focus on the size-dependent beam, plate and shell models developed based on the nonlocal elasticity theory, modified couple stress theory and strain gradient theory due to their common use in predicting the global behaviour of small-scale structures. In each higher-order continuum theory, various size-dependent models based on the classical theory, first-order shear deformation theory and higher-order shear deformation theory were reviewed and discussed. In addition, the development of finite element solutions for size-dependent analysis of beams and plates was also highlighted. Finally a summary and recommendations for future research are presented. It is hoped that this review paper will provide current knowledge on the development of higher-order continuum models and inspire further applications of these models in predicting the behaviour of micro- and nano-structures.

Keywords: Size effect; nonlocal elasticity theory; modified couple stress theory; strain gradient theory

1. Introduction

Small-scale structural elements such as beams, plates and shells are commonly used as components in micro- and nano-electromechanical systems (MEMS and NEMS), sensors, actuators and atomic force microscopes. In these applications, size effects were experimentally observed in mechanical properties [1-5]. These size effects can be captured using either molecular dynamics (MD) simulations or higher-order continuum mechanics. Although the MD method can provide accurate predictions, it is too computationally expensive. Therefore, higher-order continuum mechanics approach was widely

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used in the modelling of small-scale structures.

The development of higher-order continuum theories can be traced back to the earliest work of Piola on the 19th century as demonstrated in [6-7] and the work of Cosserat and Cosserat [8] in 1909. However, until 1960s, the Cosserat brothers' idea was received considerable attention from researchers, and a large number of higher-order continuum theories have been developed. In general, these theories can be categorized into three different classes namely the strain gradient family, microcontinuum and nonlocal elasticity theories. The strain gradient family is composed of the couple stress theory, the first and second strain gradient theory, the modified couple stress theory and the modified strain gradient theory. In the strain gradient family, both strains and gradient of strains are considered in the strain energy, and thus the size effect is accounted for using material length scale parameters. In the couple stress theory initiated by Toupin [9], Mindlin and Tiersten [10] and Koiter [11], only the gradient of rotation vector is considered in the strain energy, and thus only two material length scale parameters are required. The modified couple stress theory was proposed by Yang et al. [12] based on modifying the couple stress theory. By introducing an equilibrium condition of moments of couples to enforce the couple stress tensor to be symmetric, the number of material length scale parameters of the modified couple stress theory is reduced from two to one. The first strain gradient theory initiated by Mindlin [13] considers only the first gradient of strains. One year later, Mindlin [14] derived the second strain gradient theory which is considered as the most general form of strain gradient theory accounting for both the first and second gradients of strains. Lam et al. [15] proposed the modified strain gradient theory with only three material length scale parameters by modifying Mindlin's theory by using a similar approach of Yang et al. [12]. The microcontinuum theory was developed by Eringen [16-18] consisting of micropolar, microstretch and micromorphic (3M) theories. Micropolar theory which is actually initiated by Cosserat brothers [8] is the simplest one among 3M theories, whilst micromorphic theory is the most general one among 3M theories. In 3M theories, each particle can rotate and deform independently regardless of the motion of the centroid of the particle. More details about the 3M theories as well as their applications can be found in [19-25]. The nonlocal elasticity theory was originally proposed by Kroner [26] and improved by Eringen [27-28] and Eringen and Edelen [29]. In this theory, the stress at a reference point in a continuum depends on the strains at all points of the body, and thus the size effect is captured by means of constitutive equations using a nonlocal parameter. Nonlocal elasticity theory was initially formulated in an integral form and later reformulated by Eringen [30] in a differential form by considering a specific kernel function. Compared to the integral model, the differential one is widely used for nanos tructures due to its simplicity. In addition, another class of higher-order theory which is called nonlocal strain gradient theory has been recently proposed based on a combination of the nonlocal elasticity theory and the strain gradient theory. The interested reader can refer to [31-33] for more details on this theory.

Size-dependent models have been widely used for predicting the global behaviour of beam- and plate-like nanostructures such as carbon nanotubes (CNTs) and graphene sheets. CNTs were discovered by Iijima [34] by rolling graphene sheets. Based on synthesis route and reaction parameters, various types of CNTs such as single-walled carbon nanotubes (SWCNTs), double-walled carbon nanotubes (DWCNTs) and multi-walled carbon nanotubes (MWCNTs) can be obtained (see Fig. 1) by rolling single-layered graphene sheets (SLGSs), double-layered graphene sheets (DLGSs) and multi-layered graphene sheets (MLGSs) (see Fig. 2). Nanotube is a key nanostructure and has a wide range of applications in all areas of nanotechnology. Notable among them is conveying fluid [35-41] and nanofluidic devices and systems.

A large number of size-dependent models have been proposed based on various beam and plate theories. The simplest models were based on Euler-Bernoulli beam theory (EBT) and classical plate theory (CPT). These models are only appropriate for modelling of slender beams and thin plates because they ignore shear deformation effect. To overcome the limitation of the EBT and CPT, a number of shear deformation theories have been proposed. First-order shear deformation models were based on Timoshenko beam theory (TBT) and first-order shear deformation theory (FSDT). Since the in-plane displacements vary linearly through the thickness in these models, a shear correction factor is required. In order to eliminate the use of the shear correction factor and obtain a better prediction of the responses of thick beams and plates, several higher-order shear deformation theories (HSDTs) have been proposed, notable among them are Reddy beam theory (RBT) and third-order shear deformation theory (TSDT) of Reddy [42]. A comprehensive review on the plate theories can be found in the work by Thai and Kim [43].

The governing equations derived from the aforementioned size-dependent models can be solved using either analytical methods or numerical approaches. However, the application of analytical methods is limited to a particular nanostructure with simple geometry, loading and boundary conditions (BCs). For instance, Navier method is only applied for rectangular plates with simply supported BCs, whilst Levy method is only applied for rectangular plates in which two opposite edges are simply supported and the remaining two edges can have any arbitrary BCs. For the practical problems with general geometry, loading and BCs, seeking their analytical solutions is impossible because of the m athematical complexity of the size-dependent models compared to the classical ones. Therefore, num erical approaches such as finite element method, differential quadrature method, mesh-free method, R itz method, Galerkin method, etc. become the most suitable ones for solving such problems. Among different numerical techniques, the finite element method is the most powerful tool and commonly used for the analysis of structures, and thus the development of finite element solutions for size-dep endent models will be discussed in this review.

Although extensive research on small-scale beams, plates and shells has been made during the pas t decade, the development of models for capturing the size effect in these structures has not been r eviewed. Therefore, this paper aims to provide a comprehensive review on the development of size-dependent models for predicting the behaviour of small-scale beam- and plate-like structures. The re view mainly focuses on the beam, plate and shell models which were developed based on the nonlo cal elasticity theory of Eringen [30], the modified couple stress theory of Yang et al. [12] and the modified strain gradient theory of Lam et al. [15]. In addition, the development of finite element m odels of these theories was also highlighted and discussed in details.

2. Nonlocal elasticity theory

2.1. Review of the nonlocal elasticity theory

The nonlocal elasticity theory was initially formulated by Eringen [27-28] and Eringen and Edelen [29] by means of integral constitutive equation as

$$\sigma_{ij} = \int_{\bar{x}} k\left(\left|x-\bar{x}\right|,\kappa\right)\sigma_{ij}^{L}dx \tag{1}$$

where σ_{ij} and σ_{ij}^{L} are the components of the nonlocal and local stress tensors, respectively and k i s the kernel function determined in terms of nonlocal parameter κ and neighbourhood distance $\left|x-\bar{x}\right|$ in which $\kappa = e_0 a$ and e_0 and a are the material constant and the internal characteristic len gth, respectively, i.e. lattice parameter, granular size or molecular diameter. The value of e_0 can be determined either from experiments or simulations. The value of e_0 was calibrated by Huang et al. [44] for the static bending analysis of SLGSs. Arash and Ansari [45] also evaluated the value of th e nonlocal parameter for the free vibration of SWCNTs by comparing the predictions from the nonl ocal FSDT shell model with MD simulations as shown in Fig. 3. Duan et al. [46] proposed a micr ostructured beam model to calibrate the value of e_0 for the free vibration analysis of nonlocal bea ms. Analytical expressions of e_0 were obtained based on geometrical properties and vibration modes. Zhang et al. [47-49] proposed a microstructured beam-grid model to determine the value of e_0 for the free vibration of nonlocal beams [47] and buckling and free vibration of nonlocal plates [48]. It was found that the value of e_0 varies with respect to initial stress, rotary inertia, mode shape and aspect ratio of rectangular plates. In general, a conservative estimate of the nonlocal parameter for SWCNTs is $e_0 a$ < 2.0 nm [50].

By considering a specific kernel function *k*, Eringen [30] reformulated the nonlocal constitutive equation in a differential form as

$$\left(1-\mu\nabla^2\right)\sigma_{ij}=\sigma_{ij}^L \tag{2}$$

where $\mu=\kappa^2$ and ∇^2 is the Laplacian operator. The explicit form of Eq. (2) can be written for three problems with isotropic materials as follows.

For one-dimensional (1D) problems:

$$\sigma_{xx}-\mu\frac{d^2\sigma_{xx}}{dx^2}=E\varepsilon_{xx} \tag{3a}$$

$$\sigma_{xz}-\mu\frac{d^2\sigma_{xz}}{dx^2}=\frac{E}{1+\nu}\varepsilon_{xz} \tag{3b}$$

For 2D problems:

$$\sigma_{xx}-\mu\left(\frac{\partial^2\sigma_{xx}}{\partial x^2}+\frac{\partial^2\sigma_{xx}}{\partial y^2}\right)=\frac{E}{1-\nu^2}\left(\varepsilon_{xx}+\nu\varepsilon_{yy}\right) \tag{4a}$$

$$\sigma_{yy}-\mu\left(\frac{\partial^2\sigma_{yy}}{\partial x^2}+\frac{\partial^2\sigma_{yy}}{\partial y^2}\right)=\frac{E}{1-\nu^2}\left(\nu\varepsilon_{xx}+\varepsilon_{yy}\right) \tag{4b}$$

$$\sigma_{xy}-\mu\left(\frac{\partial^2\sigma_{xy}}{\partial x^2}+\frac{\partial^2\sigma_{xy}}{\partial y^2}\right)=\frac{E}{1+\nu}\varepsilon_{xy} \tag{4c}$$

$$\sigma_{xz}-\mu\left(\frac{\partial^2\sigma_{xz}}{\partial x^2}+\frac{\partial^2\sigma_{xz}}{\partial y^2}\right)=\frac{E}{1+\nu}\varepsilon_{xz} \tag{4d}$$

$$\sigma_{yz}-\mu\left(\frac{\partial^2\sigma_{yz}}{\partial x^2}+\frac{\partial^2\sigma_{yz}}{\partial y^2}\right)=\frac{E}{1+\nu}\varepsilon_{yz} \tag{4e}$$

For 3D problems:

$$\sigma_{xx}-\mu\left(\frac{\partial^2\sigma_{xx}}{\partial x^2}+\frac{\partial^2\sigma_{xx}}{\partial y^2}+\frac{\partial^2\sigma_{xx}}{\partial z^2}\right)=\frac{E}{1-\nu^2}\left(\varepsilon_{xx}+\nu\varepsilon_{yy}+\nu\varepsilon_{zz}\right) \tag{5a}$$

$$\sigma_{yy}-\mu\left(\frac{\partial^2\sigma_{yy}}{\partial x^2}+\frac{\partial^2\sigma_{yy}}{\partial y^2}+\frac{\partial^2\sigma_{yy}}{\partial z^2}\right)=\frac{E}{1-\nu^2}\left(\nu\varepsilon_{xx}+\varepsilon_{yy}+\nu\varepsilon_{zz}\right) \tag{5b}$$

$$\sigma_{zz} - \mu\left(\frac{\partial^2\sigma_{zz}}{\partial x^2} + \frac{\partial^2\sigma_{zz}}{\partial y^2} + \frac{\partial^2\sigma_{zz}}{\partial z^2}\right) = \frac{E}{1-\nu^2}\left(\nu\varepsilon_{xx} + \nu\varepsilon_{yy} + \varepsilon_{zz}\right) \tag{5c}$$

$$\sigma_{xy} - \mu\left(\frac{\partial^2\sigma_{xy}}{\partial x^2} + \frac{\partial^2\sigma_{xy}}{\partial y^2} + \frac{\partial^2\sigma_{xy}}{\partial z^2}\right) = \frac{E}{1+\nu}\varepsilon_{xy} \tag{5d}$$

$$\sigma_{xz} - \mu\left(\frac{\partial^2\sigma_{xz}}{\partial x^2} + \frac{\partial^2\sigma_{xz}}{\partial y^2} + \frac{\partial^2\sigma_{xz}}{\partial z^2}\right) = \frac{E}{1+\nu}\varepsilon_{xz} \tag{5e}$$

$$\sigma_{yz} - \mu\left(\frac{\partial^2\sigma_{yz}}{\partial x^2} + \frac{\partial^2\sigma_{yz}}{\partial y^2} + \frac{\partial^2\sigma_{yz}}{\partial z^2}\right) = \frac{E}{1+\nu}\varepsilon_{yz} \tag{5f}$$

where ε_{ij} are the components of the strain tensor; and E and ν are the Young's modulus and Poisson ratio of materials, respectively.

Compared to the integral model, the differential one is widely used for nanostructures due to its simplicity. However, the differential model may give paradoxical results for certain cases, e.g. bending and vibration problems of cantilever beams. More information about paradoxical behaviour of the differential model can be found in [51-54].

2.2. Beam models

2.2.1. Nonlocal models based on the EBT

The first nonlocal beam models based on the EBT were developed by Peddieson et al. [55] and Sudak [56]. Peddieson et al. [55] applied their model to explore the size effect on the bending behaviour of isotropic nanobeams, whilst Sudak [56] applied his model to study the buckling of MWCNTs. Since the early works by Peddieson et al. [55] and Sudak [56], there have been a large number of articles devoted to the modelling of nanobeams and CNTs using the nonlocal EBT model. For example, Zhang et al. [57] investigated the free vibration of DWCNTs. Closed-form solutions for natural frequencies of simply supported DWCNTs were obtained to study the size effect on vibration characteristics of DWCNTs. Wang et al. [58] derived a general form of closed-form solutions for buckling loads of CNTs with various BCs. Aydogdu [59] examined the size effect on the axial vibration of nanorods under different BCs and obtained explicit expressions for natural frequencies. Murmu and Pradhan [60] included thermal effects in the free vibration analysis of embedded SWCNTs using the differential quadrature (DQ) method. This approach was also used by Civalek and Demir [61] to derive bending moments and deflections of nanobeams with various BCs. The free vibration of axially loaded non-prismatic embedded SWCNTs was investigated by Mustapha and Zhong [62] using the Bubnov-Galerkin method. Li et al. [63] derived closed-form solutions for natural frequencies of axially loaded simply supported nanobeams. Ghannadpour et al. [64] employed the Ritz method so lve the governing equations of the nonlocal EBT model for deflections, buckling loads and natural frequencies of nanobeams with various BCs. Based on von Karman nonlinearity, Ansari et al. [65] developed a nonlinear nonlocal EBT model for nonlinear vibrations of embedded MWCNTs in thermal environmental, whilst Fang et al. [66] developed a nonlinear nonlocal EBT model for nonlinear vibrations of embedded DWCNTs. The nonlinear free and forced vibrations of nanobeams with various BCs were examined by Simsek [67] and Bagdatli [68].

The nonlocal EBT model was also developed for nanobeams made of functionally graded (FG) materials. Simsek [69] investigated the nonlocal effect on the axial vibration of FG nanorods with variable cross-sections. The elastic modulus and mass density of nanorods were assumed to vary in the axial direction according to a power law form. Nguyen et al. [70] presented analytical solutions of the nonlocal EBT model for the static bending analysis of FG beams with various BCs. The elastic modulus of FG nanobeams can vary in either the axial direction or transverse direction. Based on the Galerkin approach, Niknam and Aghdam [71] derived solutions of the nonlocal EBT model for natural frequencies and critical buckling loads of FG nanobeams resting on an elastic foundation. Ebrahimi and Salari [72-73] studied thermal effect on the free vibration of FG nanobeams under various BCs using a semi analytical approach. Nejad and Hadi [74] and Nejad et al. [75] examined the bending [74] and buckling [75] behaviours of FG nanobeams in which the elastic modulus can vary in both axial and transverse directions of the beam. The DQ method was used to solve the governing equations for critical buckling loads of FG nanobeams with arbitrary BCs.

Nonlinear free vibration of FG nanobeams was investigated by Nazemnezhad and Hosseini-Hashemi [76] using a nonlinear nonlocal EBT model with von Karman nonlinear theory. Analytical solutions for nonlinear natural frequencies of simply supported beams were also obtained using a method of multiple scales. El-Borgi et al. [77] developed a nonlinear nonlocal EBT model for the nonlinear free and forced vibrations of FG embedded nanobeams. The method of multiple scales was employed to solve for nonlinear frequencies of simply supported beams. Shafiei et al. [78] also developed a nonlinear nonlocal EBT model to study the nonlinear free vibration of axially FG nanobeams with variable cross-sections. Nonlinear frequencies of nanobeams under various BCs were obtained using the generalized DQ technique.

2.2.2. Nonlocal models based on the TBT

The earliest nonlocal TBT model was developed by Wang [79] to study wave propagation in CNTs. The model accounts for the shear deformation effect which becomes significant in short and stocky CNTs. Wang and Varadan [80] also developed a nonlocal TBT model, but it was applied to inv estigate the free vibration of both SWCNTs and DWCNTs. Closed-form solutions for natural frequencies of simply supported CNTs were also obtained. Wang et al. [81-84] derived closed-form solutions for buckling loads [81], natural frequencies [82] and deflections [83-84] of the nonlocal TBT model with four different BCs including simply supported, clamped, cantilever and propped cantilever. In these models [81-84], the transverse shear stress was based on the local theory and thus they are inconsistent. Lu et al. [85] overcame this limitation in their consistent nonlocal TBT model in which the nonlocal effect was included in both normal and transverse shear stresses. The consistent nonlocal model was also developed by Reddy and Pang [86] by reformulating both EBT and TBT using the nonlocal constitutive relations of Eringen. Closed-form solutions for deflections, buckling loads and natural frequencies were obtained for nanobeams under four different BCs. It should be noted that the closed-form solutions derived by Reddy and Pang [86] are different with those given by Wang et al. [81-84] since they were based on different TBT models.

The consistent nonlocal TBT model has been widely used to investigate the nonlocal effect in CNTs. For example, Murmu and Pradhan [87] investigated the influences of nonlocal parameter and transverse shear deformation on the buckling of SWCNTs surrounded by an elastic medium. This work was extended by Ansari et al. [88] to include the effect of elevated temperature. Pradhan and Murmu [89] examined the nonlocal effect on the vibration of embedded SWCNTs using the consistent nonlocal TBT model and the DQ approach. Numerical solutions of the consistent TBT model were presented by Roque et al. [90] based on a meshless method with both global and local collocation techniques and radial basis functions. The vibration of embedded SWCNTs was also examined by Wu and Lai [91] using the consistent nonlocal TBT models developed based on both Reissner mixed variation theory and principle of virtual displacement. Amirian et al. [92] and Zidour et al. [93] included the thermal effect on the vibration of SWCNTs, whilst Ansari et al. [94] included the thermal effect on the dynamic stability of embedded MWCNTs. Ansari et al. [95] developed a nonlocal TBT model for the nonlinear forced vibration of magneto-electro-thermo-elastic nanobeams.

The consistent nonlocal TBT model was also developed for FG nanobeams. Simsek and Yurtcu [96] proposed both nonlocal EBT and TBT models for the bending and buckling analyses of FG nanobeams. The consistent nonlocal TBT model was extended by Rahmani and Pedram [97] to the free vibration analysis of simply supported FG nanobeams. Ebrahimi and Salari [98-99] also developed a consistent nonlocal TBT model for the buckling and free vibration analyses FG nanobeams in which thermal effects were considered.

2.2.3. Nonlocal models based on the RBT

Based on the nonlocal constitutive relations of Eringen, Reddy [100] reformulated the EBT, TBT, RBT and Levinson beam theory to include the nonlocal effect. Variational statements of four models were also derived to facilitate the development of nonlocal FE models. Closed-form solutions for deflections, buckling loads and natural frequencies were obtained for simply supported beams. Ebrahimi and Salari [101] included thermal effects in the nonlocal RBT model to examine the influences of elevated temperature and nonlocal parameter on free vibration characteristics of embedded SWCNTs. Emam [102] proposed a unified nonlinear nonlocal model for the buckling and post-buckling analyses of isotropic nanobeams. Analytical solutions for buckling load and post-buckling response were also obtained for simply supported and clamped nanobeams.

Rahmani and Jandaghian [103] extended the nonlocal RBT model to FG nanobeams. Analytical solutions for critical buckling loads were obtained for FG nanobeams under various BCs using Rayleigh-Ritz method. Ebrahimi and Barati [104] also developed a nonlocal RBT model for FG nanobeams, in which thermal effects and the interaction between the nanobeam and an elastic medium were considered.

2.2.4. Nonlocal models based on HSDTs

One of the earliest nonlocal HSDT models was developed by Aydogdu [105] for isotropic nanobeams based on the general exponential shear deformation theory of Aydogdu [106]. This theory is a general form of the exponential shear deformation theory of Karama et al. [107] (see Table 1 for the displacement field). Thai [108] also proposed a nonlocal HSDT model for isotropic nanobeams, but it was based on the refined plate theory of Shimpi [109]. The displacement field of this theory is derived based on partitioning the displacements into shear and bending parts. Tounsi et al. [110] and Zemri et al. [111] extended the nonlocal HSDT model of Thai [108] to include thermal effects [110] and non-homogeneous behaviour of FG materials [111].

Thai and Vo [112] developed a nonlocal HSDT model for isotropic nanobeams based on the sinusoidal shear deformation theory of Touratier [113], whilst Tounsi et al. [114] proposed a nonlocal quasi-3D model for isotropic nanobeams based on the quasi-3D sinusoidal theory of Thai and Kim [115] (see Table 2). It is worth noting that unlike the HSDT model, the quasi-3D model is capable of capturing the thickness stretching effect which is significant in very thick or stocky members. The extension of the sinusoidal model of Thai and Vo [112] and quasi-3D sinusoidal model of Tounsi et al. [114] to FG nanobeams was respectively made by Ahouel et al. [116] and Chaht et al. [117]. The model of Thai and Vo [112] was also employed by Pour et al. [118] and Sadatshojaei and Sadatshojaie [119] to predict nonlinear vibration responses of SWCNTs embedded in an elastic medium.

Berrabah et al. [120] compared the accuracy of various nonlocal HSDT models in predicting deflections, buckling loads and natural frequencies of isotropic nanobeams. The displacement fields of these nonlocal HSDT models were taken from the simple HSDT proposed by Thai and Choi [121] in which the in-plane and transverse displacements are divided into the bending and shear components as shown in Table 1. Ebrahimi and Barati [122] developed a unified nonlocal HSDT model for FG embedded nanobeams based on the simple HSDT of Thai and Choi [121]. The model was used to study the influences of both moisture and temperature on free vibration characteristics of FG embedded nanobeams. Mashat et al. [123] investigated the vibration and thermal buckling of embedded nanobeams under various BCs using a unified nonlocal HSDT model covering EBT, TBT, RBT and sinusoidal theory. Recently, Thai et al. [124] presented a simple nonlocal HSDT model for isotropic nanobeams which involves only one unknown. Closed-form solutions for deflections and natural frequencies were also obtained for nanobeams under various BCs. Numerical results indicated that the accuracy of the present theory is comparable with the nonlocal TBT model although it has only one unknown as in the case of the nonlocal EBT model.

2.3. Plate models

2.3.1. Nonlocal models based on the CPT

Zhang et al. [125] developed one of the earliest nonlocal shell model for the buckling analysis of MWCNTs under axial compression based on the classical shell theory. Closed-form solutions obtained for buckling loads were used to examine the nonlocal effect on the axial buckling of simply supported DWCNTs. Li and Kardomateas [126-127] developed nonlocal classical shell models to examine the thermal buckling [126] and free vibration [127] of MWCNTs. The nonlocal classical shell model was also proposed by Wang and Varadan [128] and Hu et al. [129] to investigate wave propagation in CNTs. The accuracy of the nonlocal classical shell model in predicting buckling strains of axially loaded SWCNTs was also assessed by Zhang et al. [130] by comparing with the MD simulation results as shown in Fig. 4. It can be seen that for long SWCNTs with large aspect ratios, the local EBT model can give results comparable with those obtained by nonlocal EBT model and MD simulations. However, for short SWCNTs with small aspect ratios, only nonlocal shell model can give comparable predictions by the MD simulations. Rouhi and Ansari [131] also presented a nonlocal classical shell model for axial buckling of DWCNTs under various BCs. Recently, Sarvestani [132] proposed a nonlocal classical shell model for the buckling analysis of curved MWCNTs under axial compression.

One of the earliest nonlocal plate models was developed by Lu et al. [133] based on the CPT. T he model was used to study the size effect on the bending and bucking behaviours of isotropic nanoplates. Duan and Wang [134] derived exact solutions of the nonlocal CPT model for the axisymmetric bending analysis of circular nanoplates under general loading. The effects of the nonlocal parameter on deflection, radial moment, circumferential moment and shear force of graphene circular sheets subjected to uniform loads with either clamped or simply supported BCs were examined. Aksencer and Aydogdu [135] derived Levy solutions of the nonlocal CPT model for buckling loads and natural frequencies of rectangular nanoplates with two opposite edges being simply supported and the remaining two edges having any arbitrary BCs. Shakouri et al. [136] employed the Galerkin approach to solve the governing equations of the nonlocal CPT model for natural frequencies of isotropic nanoplates under various BCs.

The nonlocal CPT model was also employed to capture the size-dependent behaviour of SLGSs and MLGSs. For example, Pradhan and Murmu [137] and Pradhan and Kumar [138] investigated the nonlocal effect on the bucking of SLGSs using the DQ method, while Babaei and Shahidi [139] and Farajpour et al. [140] examined the size effect on the buckling of quadrilateral SLGSs [139] and variable-thickness SLGSs [140] using Galerkin method. The free vibration behaviour of MLGS embedded in a polymer matrix was investigated by Pradhan and Phadikar [141]. It was found that the size effect increases when the number of layers increases. Shen et al. [142] extended the application of the nonlocal CPT model to examine the free vibration of a simply supported SLGS-based mass sensor. Ansari et al. [143] presented analytical expressions for natural frequencies of SLGSs with arbitrary BCs by considering interatomic potential in deriving material properties of SLGSs. Recently, Zhang et al. [144-146] employed the element-free kp-Ritz method to solve the governing equations of the nonlocal CPT model for natural frequencies [144], nonlinear deflections [145] and bucking loads [146] of SLGSs under various BCs.

The application of the nonlocal CPT in the above-mentioned studies was limited to graphene sheets made of isotropic materials. However, the numerical results from MD simulations carried out by Ni et al. [147] indicated that the mechanical properties of graphene sheets are anisotropic because of the hexagonal structure of the unit cells of the graphene [147]. Therefore, nonlocal orthotropic CPT models were developed to account for the effect of anisotropic mechanical properties of graphene sheets. Pouresmaeeli et al. [148] developed a nonlocal CPT model for the vibration of orthotropic DLGSs embedded in an elastic medium. Mohammadi et al. [149-150] developed a nonlocal CPT model for the free vibration analysis of orthotropic embedded SLGSs in thermal environment. Both Navier and Levy solutions for natural frequencies of rectangular SLGSs were derived. Sari and Al-Kou z [151] also presented a nonlocal CPT model for the free vibration analysis of orthotropic embedded SLGSs in which the variable thickness of SLGSs was considered. Anjomshoa [152] and Anjomshoa et al. [153] proposed nonlocal CPT models to examine the buckling [152] and free vibration [153] of orthotropic circular and elliptical SLGSs embedded in an elastic medium. The nonlocal CPT model was also developed by Mohammadi et al. [154] to examine the shear buckling of orthotropic embedded SLGSs in thermal environment. Ashoori et al. [155] developed a nonlocal CPT model for thermal buckling of FG annular embedded nanoplates subjected to various types of thermal loads. Exact solutions for critical buckling temperature were also obtained for FG annular nanoplates with clamped BCs.

2.3.2. Nonlocal models based on the FSDT

The earliest nonlocal FSDT model was developed by Lu et al. [133] for isotropic nanoplates. The model was then applied to study the size effect on deflections and natural frequencies of simply supported isotropic nanoplates. Pradhan and Phadikar [156-157] presented both nonlocal CPT and FSDT models for the free vibration [156] and buckling analysis [157] of SLGSs and MLGSs. In the MLGS models, the interaction between two graphene sheets was modelled by Winkler foundation. The influences of small-scale, shear deformation, elastic modulus and stiffness of Winkler foundation on natural frequencies and critical buckling loads of simply supported graphene sheets were also investigated. Kananipour [158] also presented both nonlocal CPT and FSDT models for graphene sheets, but they were applied to the static bending analysis of DLGSs under various BCs using the DQ method. Ansari et al. [159-160] examined the vibration of SLGSs [159] and MLGSs [160] with different BCs using the nonlocal FSDT model and DQ method. The nonlocal FSDT was also employed by Samaei et al. [161] and Bedroud et al. [162] to examine the buckling of embedded SLGSs [161] and circular nanoplates [162]. Arani et al. [163] examined electro-thermal-torsional buckling of simply supported embedded double-walled boron nitride nanotubes based on the nonlocal FSDT shell model. Naderi and Saidi [164] modified the nonlocal FSDT model for buckling of nanoplates by eliminating the nonlocal effect for the transverse shear stresses. The nonlinear nonlocal CPT and FSDT models were developed by Reddy [165] for the nonlinear bending analysis of isotropic nanoplates based on von Karman nonlinearity. The variational statement of these models was also presented for the development of finite element solutions.

The nonlocal FSDT models were also proposed for nanoplates made of FG and orthotropic materials. For example, Hosseini-Hashemi et al. [166] developed a nonlocal FSDT for FG circular nanoplates. Closed-form solutions for natural frequencies of circular nanoplates under various BCs were also obtained. Anjomshoa and Tahani [167] developed a nonlocal FSDT model for the free vibration analysis of orthotropic circular and elliptical SLGSs embedded in an elastic foundation. Golmakani and Rezatalab [168] presented a nonlinear nonlocal FSDT model for the nonlinear bending analysis of orthotropic embedded SLGSs using von Karman nonlinear strains. Recently, Dastjerdi et al. [169] and Dastjerdi and Jabbarzadeh [170] presented a nonlinear nonlocal FSDT model for the geometric nonlinear analysis of annular/circular orthotropic embedded SLGSs [169] and MLGSs [170] in which the effect of elevated temperature was considered.

2.3.3. Nonlocal models based on the TSDT

The nonlocal TSDT model was first presented by Aghababaei and Reddy [171] for isotropic nanoplates by reformulating the TSDT of Reddy [42] using the nonlocal constitutive relations of Eringen. Closed-form solutions for deflections and natural frequencies were also presented for simply supported nanoplates. This model was employed by Pradhan [172] and Pradhan and Sahu [173] to study the nonlocal effect on buckling loads [172] and natural frequencies [173] of simply supported SLGSs. The buckling of SLGSs was also examined by Ansari and Sahmani [174] using a unified nonlocal model representing three different theories of the CPT, FSDT and TSDT. Hosseini-Hashemi et al. [175] derived Levy solutions for critical buckling loads and natural frequencies of isotropic nanoplates. Daneshmehr et al. [176-177] extended the application of the nonlocal TSDT to the buckling [176] and free vibration analysis [177] of FG nanoplates.

2.3.4. Nonlocal models based on HSDTs

Narendar [178] proposed a nonlocal HSDT model for the buckling analysis of isotropic nanoplates based on the refined plate theory of Shimpi [109]. This model was extended by Malekzadeh and Shojaee [179] and Narendar and Gopalakrishnan [180] to the free vibration analysis of nanoplates [179] and buckling analysis of orthotropic nanoplates [180]. This model was also employed by Sobhy [181] to examine the free vibration of orthotropic DLGSs under hydrothermal conditions. Sobhy [182] presented a general HSDT model for MLGSs based on the simple HSDTs of Thai and Choi [121] (see Table 1). Analytical solutions for natural frequencies, buckling loads and buckling temperatures were also obtained for MLGSs under various BCs. Levy solutions of the nonlocal HSDT model of Narendar [178] were derived by Sobhy [183-184] for the bending analysis of isotropic SLGSs in thermal environment [183] and orthotropic nanoplates in a hygrothermal environment [184]. Zenkour and Sobhy [185], Alzahrani et al. [186], Thai et al. [187] and Sobhy [188-189] developed nonlocal sinusoidal models for thermal buckling of embedded nanoplates [185], hydro-thermal-mechanical bending of nanoplates [186], isotropic nanoplates [187], embedded SLGSs [188] and orthotropic embed ded nanoplates [189] based on the sinusoidal theory of Touratier [113]. It is noted that the nonlocal sinusoidal model developed by Sobhy [190] for FG embedded nanoplates was based on the simple sinusoidal theory of Thai and Vo [191], and thus it is simpler than the nonlocal sinusoidal models proposed in [185-189]. Belkorissat et al. [192] also developed a simple nonlocal HSDT model for FG nanoplates which is similar to the work of Sobhy [190], but it was based on the hyperbolic function of Soldatos [193]. Khorshidi and Fallah [194] reformulated the exponential theory of Karama et al. [107] for FG nanoplates. Bessaim et al. [195] developed a nonlocal quasi-3D model for the free vibration analysis of isotropic nanoplates based on the quasi-3D sinusoidal theory of Thai and Kim [115] which involves five unknowns as shown in Table 2. Recently, Sobhy and Radwan [196] also developed a nonlocal quasi-3D theory for the free vibration and buckling of FG nanoplates. The theory has five unknowns and is similar with the one proposed in [195], but it is based on a new hyperbolic function as shown in Table 2.

3. Modified couple stress theory

3.1. Review of the modified couple stress theory

The modified couple stress theory was proposed by Yang et al. [12] by modifying the classical couple stress theory of Toupin [9], Mindlin and Tiersten [10] and Koiter [11]. By introducing an additional equilibrium condition of moments of couples to enforce the couple stress tensor to be symmetric, the number of additional material length scale parameters in the modified couple stress theory is reduced from two to one. This makes the modified couple stress theory more advantageous because the determination of the material parameters is a challenging task. The strain energy U is a function of both strain and curvature as [12]

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)dV \tag{6}$$

where m_{ij} are the components of the deviatoric part of the symmetric couple stress tensor; and χ_{ij} are the components of the symmetric curvature tensor defined by

$$\chi_{xx} = \frac{\partial\theta_x}{\partial x} \tag{7a}$$

$$\chi_{yy} = \frac{\partial\theta_y}{\partial y} \tag{7b}$$

$$\chi_{zz} = \frac{\partial\theta_z}{\partial z} \tag{7c}$$

$$\chi_{xy} = \frac{1}{2}\left(\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right) \tag{7d}$$

$$\chi_{xz} = \frac{1}{2}\left(\frac{\partial\theta_x}{\partial z} + \frac{\partial\theta_z}{\partial x}\right) \tag{7e}$$

$$\chi_{yz} = \frac{1}{2}\left(\frac{\partial\theta_y}{\partial z} + \frac{\partial\theta_z}{\partial y}\right) \tag{7f}$$

where the rotation vector θ is defined in terms of the displacement field (u_x, u_y, u_z) as

$$\theta_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) \tag{8a}$$

$$\theta_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) \tag{8b}$$

$$\theta_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \tag{8c}$$

For a linear elastic material, m_{ij} are given as

$$m_{ij} = \frac{E}{1+\nu}\ell^2 \chi_{ij} \tag{9}$$

where ℓ is the material length scale parameter. The evaluation and calibration of ℓ can be found in Refs. [197-198].

3.2. Beam models

3.2.1. Modified couple stress models based on the EBT

The earliest modified couple stress EBT model was developed by Park and Gao [199] for isotropic microbeams. They utilized their model to investigate the effect of the material length scale parameter on the deflection and bending rigidity of a cantilever epoxy beam subjected to a concentrated load at the free end. It was found that the inclusion of the material length scale parameter leads to an increase in the bending rigidity of the cantilever microbeam. This effect becomes significant when the beam thickness is small, but it is negligible with the increase of the beam thickness. This observation agrees well with the experimental data. The modified couple stress EBT model was extended by Kong et al. [200-201] to the free vibration [200] and buckling [201] problems of isotropic microbeams.

The nonlinear modified couple stress EBT model was first developed by Xia et al. [202] for the nonlinear bending, post-buckling and nonlinear free vibration analyses of isotropic microbeams based on von Karman nonlinearity. The results indicated the importance of considering nonlinearity and size effects in the proper design of microscale devices and systems such as biosensors, atomic force microscopes and MEMS [202]. Simsek [203] also developed a nonlinear EBT model for the nonline ar bending and vibration analyses of isotropic microbeams accounting for the interaction between the beam and an elastic medium. The nonlinear EBT model was widely used to study the size effect on the nonlinear bending [204], nonlinear vibration [205-209] and post-buckling [208-209] responses of isotropic microbeams. It is worth noting that Farokhi et al. [206] considered initial geometric imperfections in the nonlinear forced vibration of beams, whilst Togun and Bagdatli [207] included the axial pretension in the nonlinear free vibration of microbeams. Meanwhile, Wang et al. [204, 208] accounted for thermal effect in the nonlinear bending [204], post-buckling and free vibration [208] of beams. Ansari et al. [209] derived closed-form solutions for the vibration and post-buckling analyses of microbeams under various BCs.

The EBT model was also applied to the FG microbeams. For example, it was employed by Asghari et al. [210] to predict the bending and free vibration behaviours of FG microbeams. Free vibration of FG tapered microbeams with material properties varying in the longitudinal direction was examined by Akgoz and Civalek [211] and Shafiei et al. [212]. It should be noted that Akgoz and Civalek [211] only considered cantilever beams, whilst Shafiei et al. [212] included the geometric nonlinearity in beams with different BCs. Simsek [213] also examined the nonlinear free vibration of axially FG microbeams. However, he used the Galerkin and He's variational method to obtain approximate solutions for the beams with simply supported and clamped BCs. Dehrouyeh-Semnani et al. [214] included initial geometric imperfections on the free vibration analysis of FG microbeams.

3.2.2. Modified couple stress models based on the TBT

Ma et al. [215] first developed the modified couple stress TBT model by extending the EBT model of Park and Gao [199] to account for the shear deformation effect. The model was employed to investigate the effects of the material length scale parameter and shear deformation on deflections and natural frequencies of simply supported isotropic microbeams. Closed-form solutions of the TBT model were derived by Asghari et al. [216] for bending response of the beams under various BCs, whilst approximate solutions of the TBT model were derived by Dos Santos and Reddy [217] for buckling loads and natural frequencies of the beams with various BCs using the Ritz method. Dehrouyeh-Semnani and Nikkhah-Bahrami [218] used both EBT and TBT models to examine the Poisson effect in isotropic microbeams. By comparing the bending rigidities and deflections of epoxy cantilever microbeams under concentrated loads predicted by the modified couple stress models and experimental tests, it was found that the inclusion of the Poisson effect in the modified couple stress models leads to underestimating the deflection of the epoxy cantilever microbeam as shown in Fig. 5. Liu and Reddy [219] developed a modified couple stress TBT model for isotropic curved microbeams, and applied it to bending and free vibration problems of simply supported curved beams. Taati et al. [220] also developed a TBT model to investigate thermal effects in isotropic microbeams. Asghari et al. [221] presented a nonlinear TBT model for the bending and free vibration analyses of isotropic microbeams. Ghayesh et al. [222-223] also presented a nonlinear TBT model for nonlinear resonant problems of isotropic microbeams.

The TBT model was also applied to microbeams made of FG and laminated composite materials. Reddy [224] developed both EBT and TBT models for FG microbeams considering geometric nonlinearity. Closed-form expressions for deflections, buckling loads and natural frequencies of simply supported microbeams were also given. Ke et al. [225] developed a nonlinear TBT model to examine the size effect on nonlinear vibration characteristics of FG microbeams. Asghari et al. [226] developed a TBT model to investigate the size effect on the deflections and rotations of cantilever FG beams as well as on the natural frequencies of simply supported FG beams. However, the geometric nonlinearity was ignored in their model. Ke and Wang [227] utilized the TBT model to study the free vibration, static buckling and dynamic stability behaviours of FG microbeams under different BCs using the DQ method. The buckling and free vibration responses of FG microbeams at elevated temperature were investigated by Nateghi and Salamat-talab [228] using the modified couple stress TBT model. The DQ method was employed to obtain critical buckling loads and natural frequencies of FG microbeams with various BCs. Numerical results indicated that the effect of temperature becomes more significant at higher values of the ratio of the beam thickness to material length scale parameter. Simsek et al. [229] adopted the TBT model to investigate the size effect on deflections of simply supported FG microbeams subjected to uniform and concentrated loads. The application of the TBT model was extended by Chen et al. [230], Chen and Li [231], Roque et al. [232] and Mohammad-Abadi and Daneshmehr [233] to the static bending [230, 232], free vibration [231] and buckling [233] of laminated composite microbeams. Thai et al. [234] also extended the application of the TBT model to static bending, buckling and free vibration problems of FG sandwich microbeams. Recently, Krysko et al. [235] developed a TBT model for the static bending and free vibration analyses of three layer microbeams based on Grigolyuk-Chulkov theory.

3.2.3. Modified couple stress models based on the RBT

The modified couple stress RBT model was first proposed by Ma et al. [236] for isotropic microbeams. It was used to examine the size effect on the static bending and free vibration responses of simply supported microbeams. The application of the RBT model was extended by Mohammad-Abadi and Daneshmehr [237] to investigate the size effect on buckling behaviour of isotropic microbea ms. Both EBT and TBT models were also included in their works. Analytical solutions for simply supported beams were also provided for a comparison purpose.

Salamat-talab et al. [238] extended the application of the RBT model to FG microbeams, and derived closed-form solutions for deflections and natural frequencies of simply supported microbeams. Nateghi et al. [239] and Aghazadeh et al. [240] presented a unified model for buckling [239], bending and free vibration [240] of FG microbeams. The unified model covers three different beam theories of the EBT, FBT and TBT. The DQ solution method is used to solve for the buckling loads, deflections and natural frequencies of FG microbeams under different BCs. Chen et al. [241] developed a RBT model for laminated composite microbeams based on a new constitutive relation for anisotropic materials. The model was used to examine the size effect on deflections of cross-ply simply supported microbeams under uniform loads. Mohammad-Abadi and Daneshmehr [242] and Mohammad-Abadi et al. [243] extended their isotropic model in [237] to study the free vibration [242] and thermal buckling [243] of laminated composite microbeams under various BCs.

3.2.4. Modified couple stress models based on HSDTs

Darijani and Mohammadabadi [244] proposed a modified couple stress HSDT model for isotropic microbeams by separating the axial and transverse displacements into the shear and bending parts. The shape function of the shear part as shown in Table 1 was determined based on the condition that both transverse shear stress and couple stress vanish on the top and bottom surfaces of the cross-section. Recently, Noori et al. [245] presented a HSDT model for free vibration of isotropic microbeams based on a fifth-order variation of the axial displacement across the thickness. The DQ solution method was employed to solve for natural frequencies of microbeams under various BCs. Simsek and Reddy [246] developed a unified HSDT model for FG microbeams covering seven different beam theories including EBT, TBT, RBT, sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193], exponential theory of Karama et al. [107] and general exponential theory of Aydogdu [106]. The model was applied to the bending and free vibration problems of simply supported FG microbeams. The model was also extended by Simsek and Reddy [247] and Akbarzadeh Khorshidi et al. [248] to buckling problems of FG embedded microbeams [247] and post-buckling problems of FG microbeams with general BCs [248]. Trinh et al. [249] also presented a unified modified couple stress model for FG microbeams composed of both HSDT and quasi-3D theories of beams. The displacement field of their model was based on that proposed by Thai et al. [250] in which the transverse displacements are partitioned into bending, shear and thickness stretching components as shown in Table 2.

Based on the sinusoidal theory of Touratier [113], Akgoz and Civalek [251] developed a modified couple stress sinusoidal model to investigate thermal-mechanical buckling characteristics of simply supported FG embedded microbeams. The results indicated that the effect of elevated temperature on buckling loads of FG microbeams becomes significant when the ratio of the thickness to material length scale parameter increases [251]. Al-Basyouni et al. [252] also presented a modified couple stress sinusoidal model for FG microbeams. However, it was based on the simple sinusoidal theory proposed by Thai and Vo [191], and included the physical neutral surface of FG microbeams.

3.3. Plate models

3.3.1. Modified couple stress models based on the CPT

The modified couple stress CPT model was first proposed by Tsiatas [253] for the bending analysis of isotropic microplates with arbitrary shape. This model was extended by Yin [254] and Jomehzadeh et al. [255] for the free vibration analysis of simply support microplates [254] and Levy-type microplates [255]. Akgoz and Civalek [256] proposed a modified couple stress theory CPT model to investigate the size effect on the free vibration of simply supported SLGSs embedded in an elastic matrix. It was found that the size effect becomes remarkable for higher modes of vibration. Akgoz and Civalek [257] also included the elastic medium in CPT model for the static bending, buckling and free vibration analysis of isotropic microplates. Askari and Tahani [258] derived closed-form solutions for natural frequencies of clamped CPT microplates using extended Kantorovich method. Simsek et al. [259] adopted the CPT model to examine the size effect on the forced vibration of isotropic microplates under a moving load. The dynamic responses of microplates under various BCs were obtained using the implicit time integration method of Newmark. Zhou et al. [260] developed a modified couple stress shell model for the free vibration analysis of isotropic microshells based on the classical shell theory. It was found that the size effect becomes remarkable when the characteristic radius size is comparable to the material length scale parameter [260].

Asghari [261] proposed a nonlinear modified couple stress CPT model for the geometrically nonlinear analysis of microplates with arbitrary shapes. Wang et al. [262-263] developed a nonlinear modified couple stress CPT model to investigate the size effect on the nonlinear free vibration [262] and nonlinear bending responses [263] of circular microplates. Farokhi and Ghayesh [264] also developed a nonlinear modified couple stress CPT model for the nonlinear dynamic analysis of isotropic microplates including initial geometric imperfections.

In addition to the application to isotropic microplates, the modified couple stress CPT model was also applied to FG microplates. Ke et al. [265] studied the size effect on deflections, critical buckl ing loads and natural frequencies of FG annular microplates under different BCs. Asghari and Taati [266] investigated free vibration of FG microplates with arbitrary shapes. Ashoori and Sadough Vanini [267] presented a modified couple stress CPT model for the buckling analysis of FG microplates which included thermal effects and the interaction between the plate and an elastic medium. Recently, Ashoori and Sadough Vanini [268] extended their work [267] to account for geometric nonlinearity on thermal buckling of circular FG microplates. Taati [269] derived analytical solutions of the nonlinear modified couple stress CPT model for buckling and post-buckling loads of FG microplates with various BCs subjected to in-plane shear, biaxial compression and uniformly transverse loads. Based on the classical shell theory, Beni et al. [270] developed a modified couple stress shell model to investigate the size effect on natural frequencies of simply supported FG cylindrical microshells. Tsiatas and Yiotis [271] developed a modified couple stress CPT model to investigate the size effect on the static bending, buckling and free vibration responses of skew microplates. By comparing with the nonlocal CPT mode, it was found that the effect of the material length scale parameter on critical buckling loads and natural frequencies is in contradiction with that of the nonlocal parameter of the nonlocal model.

3.3.2. Modified couple stress models based on the FSDT

One of the earliest modified couple stress FSDT models was developed by Ma et al. [272] and Ke et al. [273] for isotropic microplates. It is worth noting that the FSDT model of Ma et al. [272] considering both stretching and bending deformations, whilst Ke et al. [273] considered only bending deformation in their model. In addition, Ma et al. [272] derived closed-form solutions for bending and free vibration problems of simply supported plates, whilst Ke et al. [273] derived numerical solutions for natural frequencies of plates with simply supported and clamped BCs using the p-version Ritz method. Roque et al. [274] presented numerical solutions of the modified couple stress FSDT model for the static bending analysis of isotropic microplates using the meshless collocation method with radial basis functions. Zhou and Gao [275] developed a modified couple stress FSDT model for the axisymmetric bending analysis of isotropic circular microplates. Recently, Alinaghizadeh et al. [276] developed a modified couple stress FSDT model for static bending analysis of FG annular sector microplates. The DQ solution method was used to solve for deflection of microplates under various BCs. He et al. [277] extended the FSDT model to the static bending analysis of laminated composite skew microplates, whilst Simsek and Aydın [278] extended the FSDT model to the static bending and forced vibration analysis of FG microplates under a moving load.

Reddy and Berry [279] extended the axisymmetric FSDT model of Zhou and Gao [275] to accou nt for the influences of geometric nonlinearity, elevated temperature and non-homogeneous behaviour of FG materials on the axisymmetric nonlinear bending analysis of circular microplates. Ke et al. [280-281] also developed an axisymmetric FSDT model for the nonlinear free vibration [280] and post-buckling analysis [281] of FG annular microplates. Thai and Choi [282] proposed nonlinear CPT and FSDT models for FG microplates. Analytical expressions for linear and nonlinear deflections, buckling loads and natural frequencies of simply supported microplates were derived to explore the size effect on the bending, buckling and vibration responses of FG microplates. Jung et al. [283-284] included the interaction between the plate and an elastic medium in the FSDT model in investigating the size effect on the bending, vibration [283] and buckling responses [284] of simply supported FG microplates. The nonlinear FSDT models were also developed by Ansari et al. [285-286] for the nonlinear vibration [285], nonlinear bending and post-buckling analysis [286] of FG microplates. It is noted that the nonlinear FSDT model developed in [286] considered the physical neutral surface of FG plates and thus the stretching-bending coupling was eliminated. Ansari et al. [287] adopted the nonlinear FSDT model to investigate the size effect on the post-buckling path and frequency of FG microplates. Lou and He [288] also presented nonlinear CPT and FSDT models for the nonlinear bending and free vibration analysis of FG microplates. The interaction between the plate and an elastic medium, and the physical neutral surface of FG plates were taken into account in their models.

Based on the FSDT, Zeighampour and Beni [289] and Hosseini-Hashemi et al. [290] presented shell models for the free vibration analysis of isotropic cylindrical microshells [289] and spherical microshells [290]. Gholami et al. [291] also developed a FSDT shell model, but it was applied to the axial buckling and dynamic stability of FG microshells. Tadi Beni et al. [292] presented a FSDT shell model for FG cylindrical microshells, and applied to the free vibration problems. Lou et al. [293] developed a nonlinear FSDT shell model to examine the influence of the pre-buckling deformation and material length scale parameter on critical buckling loads of FG cylindrical microshells. The physical neutral surface of FG shells was considered in their model. It should be noted that the work in [293] is more advanced than that by Gholami et al. [291] since the von Karman nonlinearity and the pre-buckling deformation were taken into consideration.

3.3.3. Modified couple stress models based on the TSDT

The modified couple stress TSDT model was first developed by Gao et al. [294] for isotropic plates. The model was employed to examine the size effect on deflections and natural frequencies of simply supported microplates. This model was extended by Thai and Kim [295] and Chen et al. [296]

to microplates made of FG [295] and laminated composite materials [296]. Jung and Han [297] also presented a TSDT model for FG microplates, but they used a different law to compute the equivalent mechanical properties of FG microplates. Eshraghi et al. [298] developed a TSDT model for FG microplates with annular and circular shapes. The displacement field was expressed in a unified form representing three different plate theories of CPT, FSDT and TSDT. The DQ solution method was employed to solve for static bending and free vibration problems. Eshraghi et al. [299] recently extended their previous work [298] to include thermal effects. The nonlinear TSDT model was developed by Ghayesh and Farokhi [300] to examine nonlinear vibration characteristics of isotropic microplates. Based on the TSDT, Sahmani et al. [301] developed a modified couple stress shell model for the dynamic instability analysis of FG cylindrical microshells. Closed-form solutions were also obtained for simply supported cylindrical microshells.

3.3.4. Modified couple stress models based on HSDTs

Thai and Vo [302] proposed a modified couple stress HSDT model for FG microplates. The displacement field of the model was based on the sinusoidal theory of Touratier [113]. Closed-form solutions for deflections and natural frequencies were also derived for simply supported microplates. Darijani and Shahdadi [303] proposed a simple HSDT model for isotropic microplates by partitioning the displacements into the shear and bending components as shown in Table 1. The shape function of the shear component of the in-plane displacements was obtained based on the zero traction BCs of both transverse shear and couple stresses. He et al. [304] reformulated the refined plate theory of Shimpi [109] to account for size effects in FG microplates using the modified couple stress theory. The work carried out by Lou et al. [305] is similar to that conducted by He et al. [304]. However, Lou et al. [305] employed various shape functions of Thai and Choi [121]. Lou et al. [306] recently extended the work by He et al. [304] to include geometric nonlinearity and the interaction between the plate and an elastic medium. Recently, Trinh et al. [307] also presented a unified modified couple stress model for the buckling analysis FG microplates under mechanical and thermal loads based on a quasi-3D theory. The displacement field of their model was based on that proposed by Thai and Kim [115] in which the transverse displacements are partitioned into bending, shear and thickness stretching components as shown in Table 2.

Reddy and Kim [308] developed a nonlinear quasi-3D model for FG microplates accounting for thermal effects. It was based on the von Karman nonlinearity and a general quasi-3D theory which accounts for cubic and quadratic variations of the in-plane and transverse displacements across the thickness. The displacement field of the general quasi-3D theory shown in Table 2 contains 11 unkn owns. The CPT, FSDT and TSDT can be obtained from this general theory as special cases. Closed-form solutions of this model were derived by Kim and Reddy [309] for simply supported plates. Lei et al. [310] proposed a simple quasi-3D theory for the static bending and free vibration analysis of FG microplates which involves only five unknowns. The displacement field of the model has the same form of the model proposed by Thai and Kim [115] and Thai et al. [250]. However, Lei et al. [310] utilized a cubic shape function as shown in Table 2.

4. Modified strain gradient theory

4.1. Review of the modified strain gradient theory

In this theory [15], the strain energy contains two additional gradient parts of the dilatation gradient γ and the deviatoric stretch gradient η in addition to the symmetric curvature χ_{ij}. Therefore, the strain energy is written as [15]

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}\eta_{ijk} + m_{ij}\chi_{ij} \right) dV \tag{10}$$

where the symmetric curvature tensor χ_{ij} is defined in Eq. (6). The dilatation gradient vector γ_i and the deviatoric stretch gradient tensor η_{ijk} are respectively defined in Eqs. (10) and (11) as

$$\gamma_x = \frac{\partial \varepsilon_{xx}}{\partial x} + \frac{\partial \varepsilon_{yy}}{\partial x} + \frac{\partial \varepsilon_{zz}}{\partial x} \tag{11a}$$

$$\gamma_y = \frac{\partial \varepsilon_{xx}}{\partial y} + \frac{\partial \varepsilon_{yy}}{\partial y} + \frac{\partial \varepsilon_{zz}}{\partial y} \tag{11b}$$

$$\gamma_z = \frac{\partial \varepsilon_{xx}}{\partial z} + \frac{\partial \varepsilon_{yy}}{\partial z} + \frac{\partial \varepsilon_{zz}}{\partial z} \tag{11c}$$

$$\eta_{xxx} = \frac{\partial \varepsilon_{xx}}{\partial x} - \frac{1}{5}\left[\gamma_x + 2\left(\frac{\partial \varepsilon_{xx}}{\partial x} + \frac{\partial \varepsilon_{xy}}{\partial y} + \frac{\partial \varepsilon_{xz}}{\partial z} \right) \right] \tag{12a}$$

$$\eta_{yyy} = \frac{\partial \varepsilon_{yy}}{\partial y} - \frac{1}{5}\left[\gamma_y + 2\left(\frac{\partial \varepsilon_{xy}}{\partial x} + \frac{\partial \varepsilon_{yy}}{\partial y} + \frac{\partial \varepsilon_{yz}}{\partial z} \right) \right] \tag{12b}$$

$$\eta_{zzz} = \frac{\partial \varepsilon_{zz}}{\partial z} - \frac{1}{5}\left[\gamma_z + 2\left(\frac{\partial \varepsilon_{xz}}{\partial x} + \frac{\partial \varepsilon_{yz}}{\partial y} + \frac{\partial \varepsilon_{zz}}{\partial z} \right) \right] \tag{12c}$$

$$\eta_{yyx} = \eta_{yxy} = \eta_{xyy} = \frac{1}{3}\left[\eta_{xxx} + 2\frac{\partial \varepsilon_{xy}}{\partial y} + \frac{\partial \left(\varepsilon_{yy} - \varepsilon_{xx} \right)}{\partial x} \right] \tag{12d}$$

$$\eta_{zzx} = \eta_{zxz} = \eta_{xzz} = \frac{1}{3}\left[\eta_{xxx} + 2\frac{\partial \varepsilon_{xz}}{\partial z} + \frac{\partial \left(\varepsilon_{zz} - \varepsilon_{xx} \right)}{\partial x} \right] \tag{12e}$$

$$\eta_{xxy} = \eta_{xyx} = \eta_{yxx} = \frac{1}{3}\left[\eta_{yyy} + 2\frac{\partial \varepsilon_{xy}}{\partial x} + \frac{\partial\left(\varepsilon_{xx} - \varepsilon_{yy}\right)}{\partial y} \right] \tag{12f}$$

$$\eta_{zzy} = \eta_{zyz} = \eta_{yzz} = \frac{1}{3}\left[\eta_{yyy} + 2\frac{\partial \varepsilon_{yz}}{\partial z} + \frac{\partial\left(\varepsilon_{zz} - \varepsilon_{yy}\right)}{\partial y} \right] \tag{12g}$$

$$\eta_{xxz} = \eta_{xzx} = \eta_{zxx} = \frac{1}{3}\left[\eta_{zzz} + 2\frac{\partial \varepsilon_{xz}}{\partial x} + \frac{\partial\left(\varepsilon_{xx} - \varepsilon_{zz}\right)}{\partial z} \right] \tag{12h}$$

$$\eta_{yyz} = \eta_{yzy} = \eta_{zyy} = \frac{1}{3}\left[\eta_{zzz} + 2\frac{\partial \varepsilon_{yz}}{\partial y} + \frac{\partial\left(\varepsilon_{yy} - \varepsilon_{zz}\right)}{\partial z} \right] \tag{12i}$$

$$\eta_{xyz} = \eta_{yzx} = \eta_{zxy} = \eta_{xzy} = \eta_{zyx} = \eta_{yxz} = \frac{1}{3}\left[\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{xz}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z} \right] \tag{12j}$$

For a linear elastic material, the higher-order stresses $\left(p_i, \tau_{ijk}, m_{ij}\right)$ are given as

$$p_i = \frac{E}{1+\nu}\ell_0^2 \gamma_i \tag{13a}$$

$$\tau_{ijk} = \frac{E}{1+\nu}\ell_1^2 \eta_{ijk} \tag{13b}$$

$$m_{ij} = \frac{E}{1+\nu}\ell_2^2 \chi_{ij} \tag{13c}$$

where ℓ_0, ℓ_1 and ℓ_2 are the material length scale parameters associated with dilatation gradient, deviatoric stretch gradient and symmetric curvature gradient, respectively.

4.2. Beam models

4.2.1. Strain gradient models based on the EBT

One of the earliest strain gradient EBT models was proposed by Kong et al. [311] to investigate the size effect on deflections and natural frequencies of isotropic cantilever microbeams. The accuracy of the strain gradient theory was also compared with that of the modified couple stress theory and classical theory as shown in Fig. 6. Comparison results indicated that the strain gradient model predicts the size effect better than the modified couple stress model since it considers additional dilatation gradient tensor and deviatoric stretch gradient tensor in addition to rotation gradient tensor. Akgoz and Civalek [312] extended the strain gradient EBT model to the buckling analysis of isotropic microbeams with cantilever and simply supported BCs. Akgoz and Civalek [313-316] also employed the strain gradient EBT model to study the size effects on the buckling of SWCNTs [313], static bending of SWCNTs [314], buckling of linearly tapered microbeams [315] and longitudinal vibration of microbeams [316].

Zhao et al. [317] developed a nonlinear strain gradient EBT model for the nonlinear bending, post-buckling and nonlinear free vibration analysis of isotropic microbeams. They highlighted the importance of including geometric nonlinearity and size effects in the proper design of microbeams. Rajabi and Ramezani [318] also developed a nonlinear strain gradient EBT model for isotropic microbeams, but applied it to static bending and free vibration problems. The nonlinear strain gradient EBT model was extended by Mohammadi and Mahzoon [319] to include temperature effects on the post-buckling of isotropic microbeams. Analytical solutions were also obtained for microbeams with various BCs. Vatankhah et al. [320] utilized the nonlinear strain gradient EBT model to examine the nonlinear forced vibration of isotropic microbeams.

Kahrobaiyan et al. [321] extended the application of the strain gradient EBT model to the bending and free vibration analysis of FG microbeams. The extension of this model to buckling problems FG microbeams was carried out by Akgoz and Civalek [322]. Closed-form solutions for critical buckling loads were also obtained for FG microbeams under various BCs. Akgoz and Civalek [323] adopted the strain gradient EBT model to examine the longitudinal free vibration of FG microbeams. Rayleigh-Ritz solution technique was used to solve for natural frequencies of FG microbeams with clamped-clamped and clamped-free BCs. Rahaeifard et al. [324] developed a nonlinear strain gradient EBT model to study the influences of geometric nonlinearity and material length scale parameters on deflections and natural frequencies of FG simply supported microbeams.

4.2.2. Strain gradient models based on the TBT

Wang et al. [325] first presented a strain gradient TBT model for the static bending and free vibration analyses of isotropic simply supported microbeams. The nonlinear strain gradient TBT models were developed by Ansari et al. [326] and Asghari et al. [327] for isotropic microbeams using von Karman nonlinearity. It is worth noting that Ansari et al. [326] applied their model for nonlinear free vibration problems, whilst Asghari et al. [327] considered both nonlinear bending and nonlinear free vibration problems in their model.

Ansari et al. [328] extended the strain gradient TBT model to FG microbeams. Closed-form solutions for natural frequencies of simply supported microbeams were derived to investigate the effects of material gradient index and small-scale on the free vibration response of FG beams. Ansari et al. [329] also extended their work [328] to free vibration of curved FG microbeams. Ansari et al. [330] developed a strain gradient TBT model for thermal buckling of FG microbeams with various BCs. Recently, Ansari et al. [331] extended the strain gradient TBT model to study linear and nonlinear vibrations of fractional viscoelastic beams. It should be noted that Gholami et al. [332] did develo p a strain gradient TBT model to examine the nonlinear pull-in stability and vibration of FG mircoswitches, but it was based on the most general form of the strain gradient theory of Mindlin [14] which is not covered in this review. The effect of temperature distributions on buckling characteristics of FG microbeams was also investigated. Xie et al. [333] employed the indirect radial basis function collocation approach to solve the EBT and TBT models for deflections, buckling loads and natural frequencies of FG microbeams under various BCs. It is noted that in the previous works dealing with FG microbeams, the material length scale parameters were assumed to be constant across the thickness. Therefore, Tajalli et al. [334] improved the previous strain gradient TBT model by accounting for the variation of the material length scale parameter across the beam thickness. Case studies on static bending and free vibration problems confirmed that the aforementioned assumption of constant material length scale parameters seems to be inaccurate [334].

The nonlinear strain gradient TBT model was developed by Ansari et al. [335] to investigate the influences of material length scale parameters and initial geometric imperfections on the post-buckling response of FG microbeams. Approximate solutions for buckling loads of FG microbeams under various BCs were also presented using the DQ method. Ansari et al. [336] extended their previous work [335] to account for thermal effects.

4.2.3. Strain gradient models based on the RBT and HSDTs

Based on the strain gradient theory of Lam et al. [15], Wang et al. [337] reformulated the RBT model to account for the size effect on the static bending and free vibration responses of isotropic microbeams. Sahmani and Ansari [338] improved the strain gradient RBT model to include thermal effects and non-homogeneous behaviour of FG materials on the buckling of FG microbeams. The strain gradient RBT model was employed by Ansari et al. [339] to explore the size effect on the free vibration of simply supported FG microbeams. Zhang et al. [340] developed a RBT model for FG embedded microbeams based on the improved RBT of Shi [341]. Sahmani et al. [342] developed a nonlinear strain gradient RBT model for nonlinear free vibration of FG microbeams.

In addition to the RBT model, the HSDT models were also proposed for strain gradient microbeams based on various HSDTs of beams such as sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193] and n-th order shear deformation theory of Xiang et al. [343] (see Table 1 for the displacement field of these theories). For example, Akgoz and Civalek [344] and Lei et al. [345] proposed strain gradient sinusoidal models for the bending and free vibration analyses of the microbeams made of isotropic materials [344] and FG materials [345] based on the sinusoidal theory of Touratier [113]. Akgoz and Civalek [346] extended their previous work [344] to buckling problems of isotropic microbeams. Akgoz and Civalek [347] also developed a strain gradient sinusoidal model for FG microbeams as in the work of Lei et al. [345]. They also proposed a new equation for calculating the shear correction factor of the TBT model. In their equation, the shear correction factor is a function of the material length scale parameters. Akgoz and Civalek [348] extended their previous work [347] to account for the interaction between the FG microbeam and an elastic medium. Based on the hyperbolic theory of Soldatos [193], Akgoz and Civalek [349] proposed a strain gradient hyperbolic model for the bending and buckling analyses of isotropic embedded microbeams. Akgoz and Civalek [350] presented a unified HSDT model for the bending analysis of simply supported embedded CNTs. The displacement field of the model was based on Simsek and Reddy [246] which covers seven beam theories including the EBT, TBT, RBT, sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193], exponential theory of Karama et al. [107] and general exponential theory of Aydogdu [106]. Zhang et al. [351] proposed a HSDT model for the bending and free vibration analyses of FG curved microbeams based on the nth-order shear deformation theory of Xiang et al. [343].

4.3. Plate models

4.3.1. Strain gradient models based on the CPT

The earliest strain gradient CPT model was developed by Wang et al. [352] for predicting size-dependent responses of isotropic microplates. A comparison between the strain gradient model and modified couple stress model as shown in Fig. 7 indicated that the first one captures the size effect better than the second one does [352]. Bending solutions of the strain gradient CPT model was solved by Ashoori Movassagh and Mahmoodi [353] for microplates under various BCs using the extended Kantorovich method, whilst buckling solutions were analytically derived by Mohammadi and Fooladi Mahani [354] for Levy-type microplates. Mohammadi et al. [355] improved their previous work [354] using exact BCs of microplates. Wang et al. [356] derived the strain gradient CPT model for the bending analysis of microplates with various BCs. Zeighampour and Tadi Beni [357], Allahbakhshi and Allahbakhshi [358], Li et al. [359], Hosseini et al. [360] and Zhang et al. [361] extended the strain gradient CPT model to SWCNTs [357], MLGSs [358], two-layered isotropic microplates [359], multi-layered orthotropic microplates [360] and isotropic embedded microplates [361].

4.3.2. Strain gradient models based on the FSDT

One of the earliest strain gradient FSDT models was proposed by Sahmani and Ansari [362] and Ansari et al. [363] for the free vibration and thermal buckling of FG microplates. Sahmani and Ansari [362] only dealt with simply supported plates, whilst Ansari et al. [363] dealt with microplates u nder various BCs using the DQ method. Ansari et al. [364] developed a nonlinear strain gradient FSDT model to examine the post-buckling of FG annular microplates under thermal loading. Ansari et al. [365] extended their previous work [363] to study the effect of elevated temperature on the bending, buckling and free vibration responses of FG microplates under various BCs. It is noted that Shenas and Malekzadeh [366] also studied the influence of elevated temperature on the free vibration of FG microplates under various BCs. However, they employed the Chebyshev-Ritz method instead of the DQ approach as in the work of Ansari et al. [365]. Ansari et al. [367] developed a FSDT model for FG circular/annular microplates under various BCs using the DQ method.

Gholami et al. [368] developed a strain gradient FSDT shell model for FG cylindrical microshells. Closed-form solutions were presented for the critical buckling load of simply supported FG cylindrical microshells under axial compression. Zhang et al. [369] also developed a strain gradient FSDT shell model for FG cylindrical microshells, but it was based on the four unknown FSDT proposed by Thai and Choi [370-372]. Therefore, their model was simpler than the one proposed by Gholami et al. [368] which involves with five unknowns.

4.3.3. Strain gradient models based on the TSDT and HSDTs

Sahmani and Ansari [362] developed a strain gradient TSDT model for the free vibration analysis of FG microplates. Closed-form solutions for natural frequencies were also presented for simply supported plates. Zhang et al. [373] developed a simple TSDT model for circular/annular FG microplates based on the simple TSDT proposed by Thai and Kim [374] which involves only four unknowns. The DQ solution method was used to solve for deflections, buckling loads and natural frequencies of circular/annular FG microplates with various BCs. Zhang et al. [375] developed a simple strain gradient HSDT model for FG microplates based on the simple HSDT proposed by Thai and Choi [376-379] which has only four unknowns. However, they included the interaction between the plate and elastic medium. Akgoz and Civalek [380] developed a strain gradient sinusoidal model for the bending, buckling and free vibration analysis of isotropic microplates based on the sinusoidal theory of Touratier [113].

5. Finite element models

5.1. Beam elements

5.1.1. Nonlocal elasticity elements

Based on a nonlocal EBT model, Eltaher and his colleagues [381-385] have developed nonlocal elements for nanobeams made of FG materials [381-383] and isotropic materials [384-385]. The EBT element has two nodes with six degrees of freedom (4-DOF) in which the axial and transverse dis placements are respectively approximated using Lagrange and Hermite cubic interpolation functions. It is noted that Eltaher et al. [381] dealt with free vibration problems of FG nanobeams, whilst Eltaher et al. [382] dealt with bending and buckling problems of FG nanobeams. Eltaher et al. [383] also dealt with free vibration characteristics of FG nanobeams, but the physical neutral surface of FG beams was taken into account in their model. Eltaher et al. [384] examined the free vibration characteristics of isotropic nanobeams, whilst Alshorbagy et al. [385] studied the static bending of isotropic nanobeams. Marotti De Sciarra [386] presented a nonlocal element for the static bending analysis of isotropic nanobeams based on the nonlocal EBT model. The element has two nodes with 6-DOF and is based on higher-order interpolation functions. Therefore, it can accurately predict the bending behaviour of nanobeams with a coarse mesh. A case study on a cantilever nanobeam under a concentrated load indicated that the nonlocal effect does exist at both left and right sides of the concentrated load. This observation is contrary to that observed from existing finite element and analytical models indicated that the nonlocal effect only exists from the location of the point load to the free end. Nguyen et al. [387] developed a nonlocal mixed element for the static bending analysis of isotropic nanobeams. The element with two nodes is C^0 continuity and is based on Lagrange interpolation functions for both deflection and bending moment. The mixed element is also capable of capturing the nonlocal effect at both sides of the concentrated load applied on a cantilever beam.

In addition to the nonlocal EBT elements reported in the above-mentioned studies, nonlocal TBT elements were also developed to capture the shear deformation effect in thick nanobeams. Reddy and El-Borgi [388] presented a complete theoretical development and finite element formulation of both nonlocal EBT and TBT models for the nonlinear bending analysis of isotropic nanobeams. Their models were based on the modified von Karman nonlinear theory which accounts for the nonlinear terms due to the transverse normal strain. The nonlinear EBT element used Lagrange and Hermite cubic interpolation functions to respectively approximate the axial and transverse displacements, whilst the nonlinear TBT element employed Lagrange interpolation functions for both axial and transverse displacements and rotation. Reddy et al. [389] extended their previous work in [388] to FG nanobeams. Eltaher et al. [390] developed a nonlocal TBT element for the static bending and buckling analysis of FG nanobeams. The element which accounts for the effect of the physical neutral surface has three nodes and is based on quadratic Lagrange interpolation functions.

5.1.2. Modified couple stress elements

Based on the modified couple stress theory and the von Karman nonlinear strains, Arbind and Reddy [391] and Arbind et al. [392] developed two-node EBT and TBT elements [391] and RBT ele ment [392] for the nonlinear bending analysis of FG microbeams. The element has 3-DOF at each node. In the nonlinear EBT element, the axial and transverse displacements were approximated using Lagrange and Hermite cubic interpolation functions, respectively. Meanwhile, the nonlinear TBT and RBT elements employed Lagrange interpolation functions for both axial displacement and rotation, and Hermite cubic interpolation functions for the transverse displacement. These models were used to study the effects of material length scale parameter and geometric nonlinearity on deflections of FG microbeams. Reddy and Srinivasa [393] also developed nonlinear two-node EBT and TBT elements for microbeams which are capable of capturing moderate rotations since they were based on the modified von Karman nonlinear theory. Unlike the von Karman nonlinear theory, the modified von Karman nonlinear theory did include the nonlinear terms due to the transverse normal strain, and thus requiring 2D constitutive relations of beams. The EBT and TBT elements developed by Arbind and Reddy [391] were employed by Dehrouyeh-Semnani and Nikkhah-Bahrami [394] to examine the size effect on the bending, buckling and free vibration responses of isotropic microbeams. Kahrobaiyan et al. [395] also developed a two-node modified couple stress TBT element for the static bending analysis of isotropic microbeams. However, their element has only 2-DOF at each node and was based on the shape functions derived by directly solving the governing equations of the modified couple stress TBT model. Numerical results indicated that the load-deflection response of a cantilever microbeams predicted by their element agrees well with the experimental result as shown in Fig. 8. The accuracy and stability of the TBT elements proposed by Kahrobaiyan et al. [395] and Arbind and Reddy [391] were assessed by Dehrouyeh-Semnani and Bahrami [396]. The results indicated that both two elements give a stable solution. However, the 6-DOF element of Arbind and Reddy [391] is more accurate than the 4-DOF element of Kahrobaiyan et al. [395] in predicting deflections of isotropic microbeams under various BCs. Recently, Karttunen et al. [397] developed an exact modified couple stress TBT element for the static analysis of FG microbeams. The element has two nodes with 3-DOF at each node. It was based on the exact shape functions derived directly from analytical solutions of the modified couple stress TBT model.

5.1.3. Strain gradient elements

Kahrobaiyan et al. [398] developed a strain gradient element for isotropic microbeams based on the EBT. The element has two nodes with 3-DOF at each node including the deflection, slope and curvature. The mass and stiffness matrices of the element were derived based on the Galerkin method with interpolation functions determined by solving directly the governing equations of the strain gradient EBT model. The element was applied to the bending analysis of a cantilever microbeam und er a concentrated force at its free end. In order to account for the shear deformation effect, Zhang et al. [399] developed a two-node strain gradient TBT element for isotropic microbeams. The element has 6-DOF at each node when considering both bending and stretching deformations, and 4-DOF at each node when considering only bending deformation. The displacement field of the element is approximated using exact hyperbolic interpolation functions derived from solving directly the governing equations of the strain gradient TBT model. Numerical results indicated that the element is capable of accurately predicting the static bending, buckling and free vibration responses of isotropic microbeams. Zhang et al. [400] also presented a strain gradient TBT element for isotropic microbeams which is similar to the one developed by Zhang et al. [399]. However, it has 4-DOF per node and considers only bending deformation. Kahrobaiyan et al. [401] developed a strain gradient TBT element for isotropic microbeams. The element has two nodes with 2-DOF at each node including the deflection and rotation. The shape functions of their element were derived by directly solving the equilibrium equations of the strain gradient TBT model with the proper BCs. By comparing with experimental results, it was concluded that the present element is capable of accurately predicting the load-deflection response of a cantilever microbeams as shown in Fig. 9. The element was successfully applied to predict the deflection and natural frequency of MEMS. It should be noted that Eltaher et al. [402], Ebrahimi et al. [403] and Ansari et al. [404-406] also developed strain gradient elements for isotropic microbeams based on EBT [402-404] and TBT [405-406] models, but they were based on the nonlocal strain gradient theory and the most general form of the strain gradient theory of Mindlin [14] which are not covered in this review.

5.2. Plate elements

5.2.1. Nonlocal elasticity elements

One of the earliest nonlocal finite element models for nanoplates was developed by Phadikar and Pradhan [407] and Ansari et al. [408] using the Galerkin method. Phadikar and Pradhan [407] developed a nonlocal CPT element for the bending, buckling and free vibration analyses of isotropic nanoplates, whilst Ansari et al. [408] proposed a nonlocal FSDT element for the free vibration analysis of MLGSs. The element developed by Phadikar and Pradhan [407] has four nodes with 3-DOF at each node and was based on Hermite cubic interpolation functions, whilst the element proposed by Ansari et al. [408] has eight nodes with 5-DOF at each node and was based on quadratic serendipity interpolation functions. Natarajan et al. [409] developed a nonlocal FSDT element for the free vibration analysis of FG nanoplates using an isogeometric analysis (IGA) in which the field variables were approximated by non-uniform rational B-splines (NURBS) basic functions as shown in Fig. 10.

Nguyen et al. [410] also employed the IGA approach to develop a nonlocal element for FG nanoplates. However, their element was based on a simple quasi-3D theory with four unknowns as shown in Table 2. Ansari and Norouzzadeh [411] studied the nonlocal and surface effects on the buckling behaviour of FG nanoplates based on the FSDT and IGA approach. Sarrami-Foroushani and Azhari [412] presented a nonlocal element for the buckling and free vibration analysis of SLGSs based on the finite strip method and the refined plate theory of Shimpi [109]. Unlike the finite element method, the plate in the finite strip approach is meshed in one direction, and thus the number of DOFs is reduced.

5.2.2. Modified couple stress elements

One of the earliest modified couple stress plate elements was developed by Zhang et al. [413] for isotropic microplates based on the FSDT. The element is non-conforming and has four nodes with 15-DOF per node. Unlike the classical FSDT element, the modified couple stress FSDT element is shear locking free and thus the full integration can still be used. The element was successfully used to predict the bending, buckling and free vibration responses of isotropic microplates with various BCs. Reddy and Srinivasa [393] presented a nonlinear FSDT element for the nonlinear analysis of modified couple stress plates. Since the element was based on the modified von Karman nonlinear theory, it is capable of capturing moderate rotations. Mirsalehi et al. [414] developed a modified couple stress CPT element for FG microplates based on a spline finite strip method. The spline finite strip method is a special form of the finite strip method in which the B3-spline functions are used in the longitudinal direction and the Hermite cubic functions are used in the transverse direction of the strip [414]. The spline finite strip element was applied to predict the critical buckling loads and buckling temperatures of FG microplates under mechanical and thermal loadings. Kim and Reddy [415] developed a nonlinear modified couple stress element for FG microplates based on the general quasi-3D theory of Reddy and Kim [308] and von Karman nonlinear strains. The element is non-conforming and has four nodes with 44-DOF at each node accounting for geometric nonlinearity and requires C^1 continuity for all variables. Reddy et al. [416] presented nonlinear CPT and FSDT elements for the axisymmetric bending analysis of circular FG microplates. The axisymmetric CPT element has two nodes with 3-DOF at each node based on Lagrange interpolation functions for the axial displacement and Hermite interpolation functions for the transverse displacement. Meanwhile, the axisymmetric FSDT element which has two nodes with 4-DOF at each node employed Lagrange interpolation functions for both axial displacement and rotation, and Hermite interpolation functions for the transverse displacement. The models were employed to study the influences of geometric nonlineari ty and material length scale parameter on bending responses of FG circular plates with various BCs. Recently, Nguyen et al. [417] proposed an efficient modified couple stress element for the static bending, buckling and free vibration analyses of FG microplates based on the IGA approach. The element was based on a simple quasi-3D theory with four unknowns as shown in Table 2.

5.2.3. Strain gradient elements

For the strain gradient plate element, only two publications were found in the literature involved in the development of the finite element model for microplates based on the strain gradient theory of Lam et al. [15]. Mirsalehi et al. [418] presented a strain gradient CPT element for FG microplates using the spline finite strip method. The element was used to investigate the influences of material length scale parameters, BCs, volume fraction module and geometric dimensions on critical buckling loads and natural frequencies of FG microplates. Recently, Thai et al. [419] developed a strain gradient element for the bending, buckling and free vibration analyses of FG microplates based on the IGA approach. It should be noted that Ansari et al. [420-421] also developed strain gradient elements for isotropic microplates, but it was based on the most general form of the strain gradient theory of Mindlin [14] which is not covered in this review.

6. Concluding remarks and recommendation for future studies

The development of size-dependent models for predicting size effects on the global responses of small-scale beam, plate and shell structures was comprehensively reviewed and discussed in this paper. During the past decade, great efforts have been devoted to the development of size-dependent models based on higher-order continuum mechanics approach. This review mainly focuses on the size-dependent beam, plate and shell models developed based on the nonlocal elasticity theory, modified couple stress theory and strain gradient theory due to their common use in predicting the global behaviour of small-scale structures. Both analytical and numerical models are included in this review paper.

The review indicates that most size-dependent models have been developed in the last five years. The number of strain gradient models is small compared to the number of models developed based on the nonlocal elasticity theory and modified couple stress theory. The nonlocal beam and plate models are widely used for analysing nanostructures such as CNTs and graphene sheets, whilst the modified couple stress and strain gradient models are applied to microstructures. The review also shows that the number of relevant papers involving in the development of finite element models is relatively small compared with the total number of papers published on analytical models.

As reviewed in this paper, most of existing size-dependent models focused on analytical solutions which are limited to beam and plate structures subjected to certain loading and boundary conditions and geometries, whereas the development of finite element solutions for size-dependent beam and plate models has not been given enough attention. Therefore, further efforts should be devoted to developing finite element models of size-dependent theories, especially the strain gradient-based models. It is noted that only one publication was found in the literature involved in the development of the finite element model for strain gradient CPT plates using the spline finite strip method.

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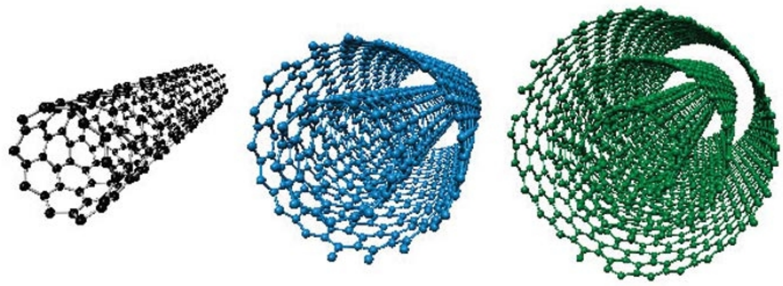
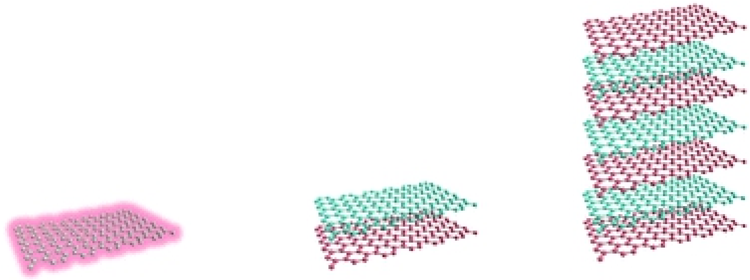


(a) SWCNT (b) DWCNT (c) MWCNT

Fig. 1 Schematic illustration of different form of CNTs [422]



(a) SLGS (b) DLGS (c) MLGS

Fig. 2 Graphene-based nanomaterials [423]

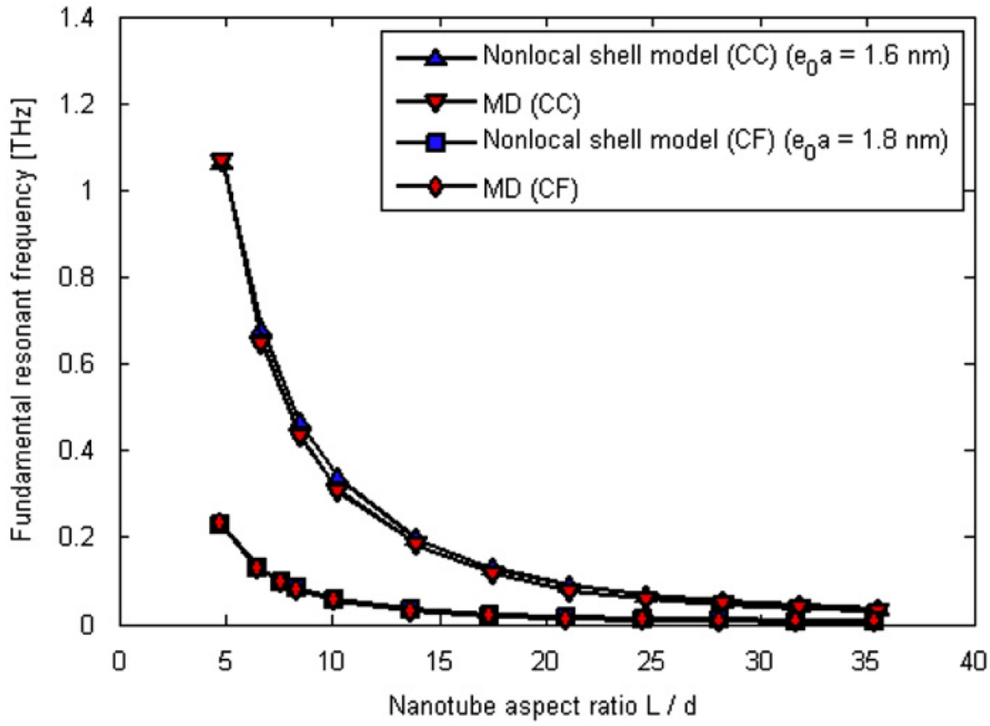


Fig. 3 Fundamental frequencies of clamped (CC) and cantilever (CF) beams [45]

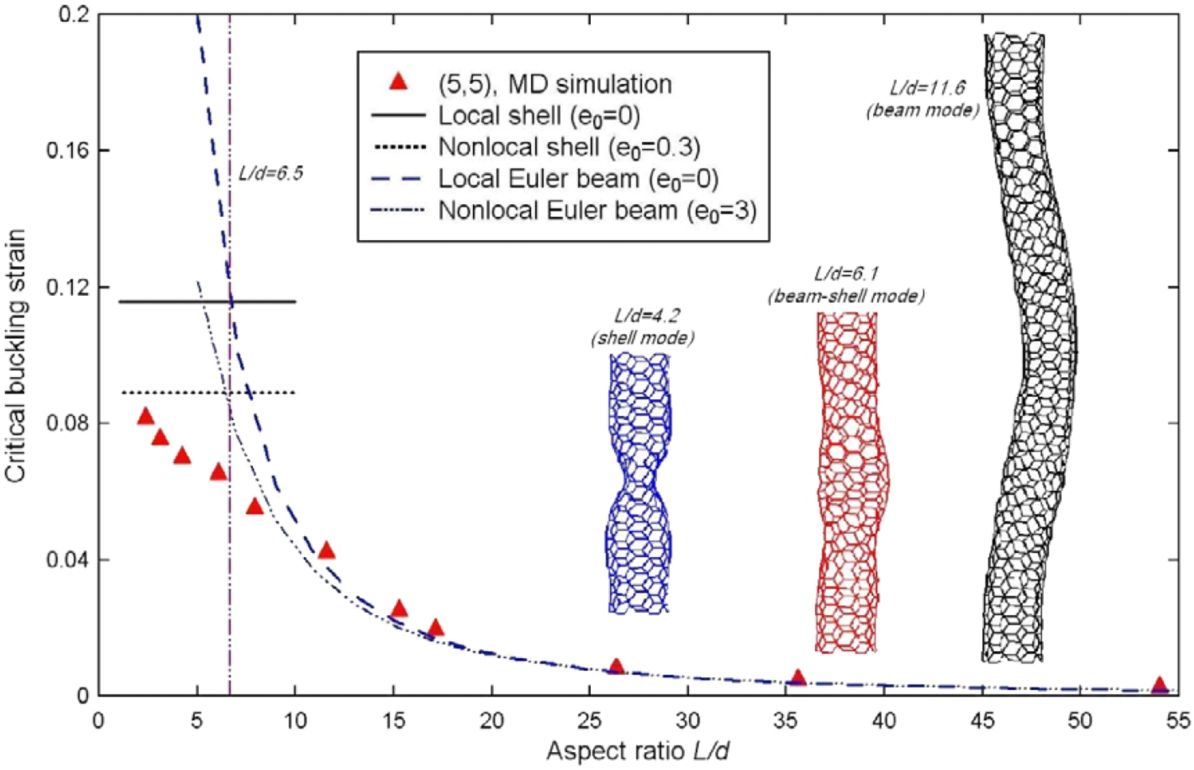
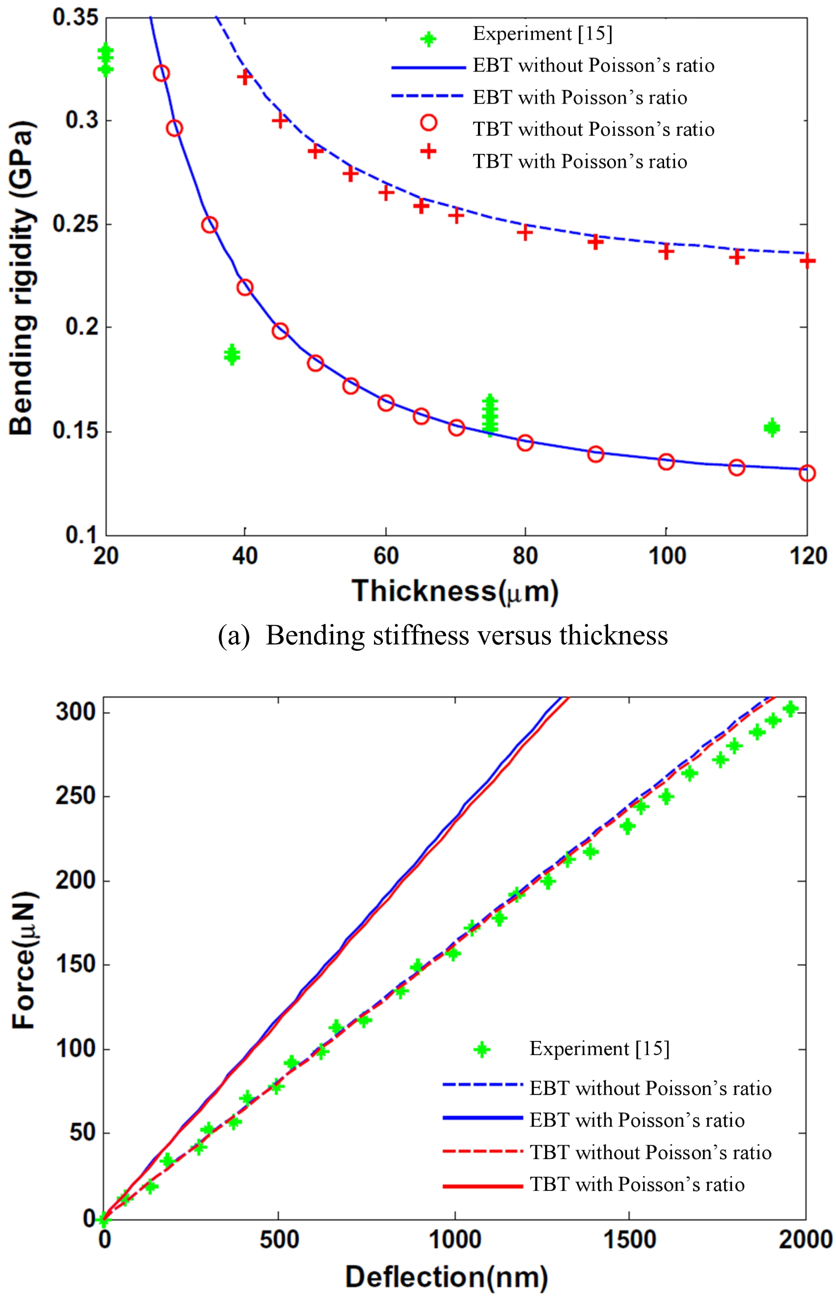


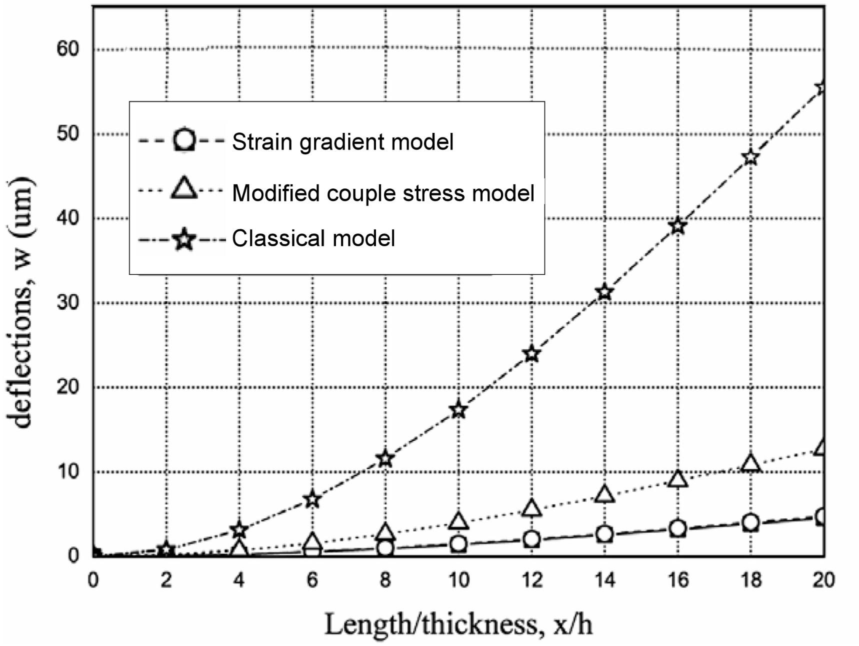
Fig. 4 Comparison of various continuum mechanics models with MD simulations for (5,5) SWCNTs [130]



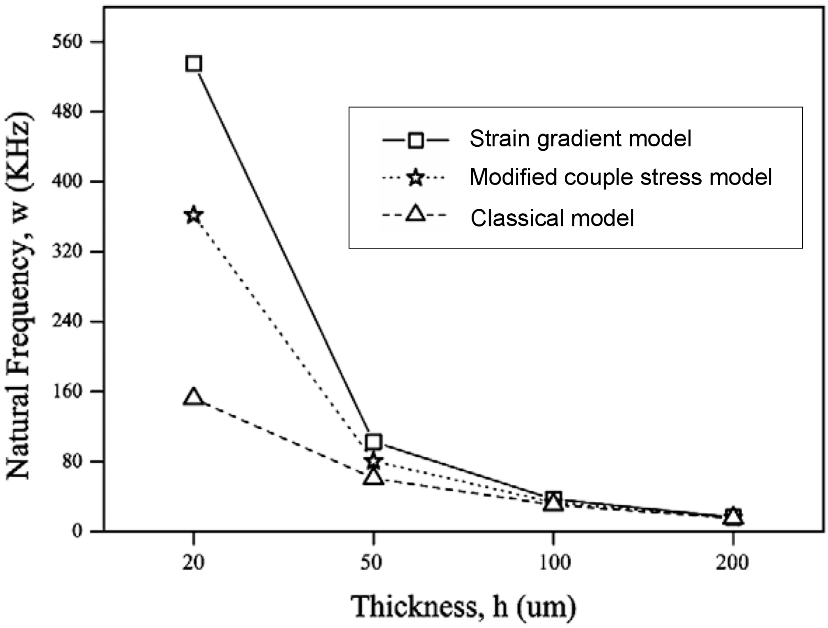
(a) Bending stiffness versus thickness

(b) Load-deflection response (h = 38 μm)

Fig. 5 Effect of Poisson's ratio in a cantilever microbeam [218]

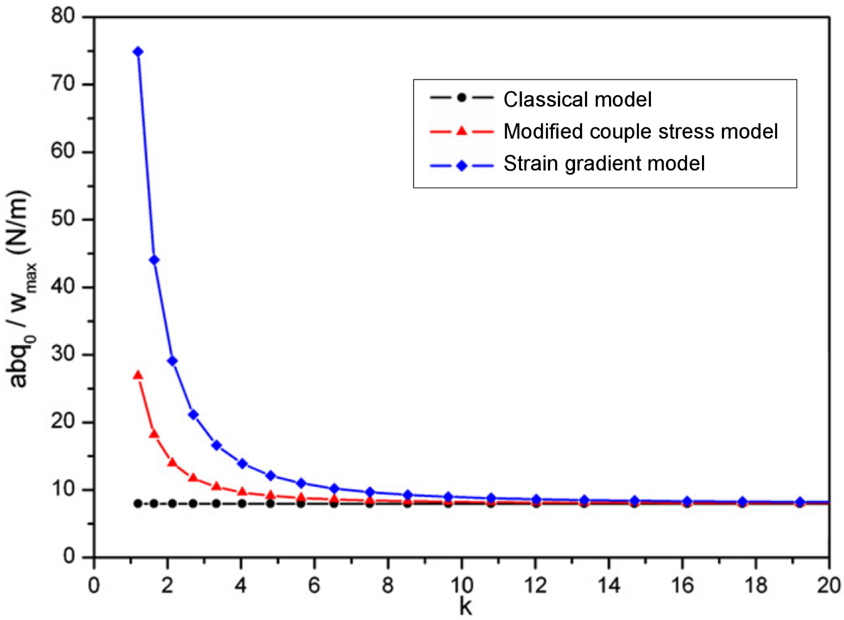


(a) Bending analysis

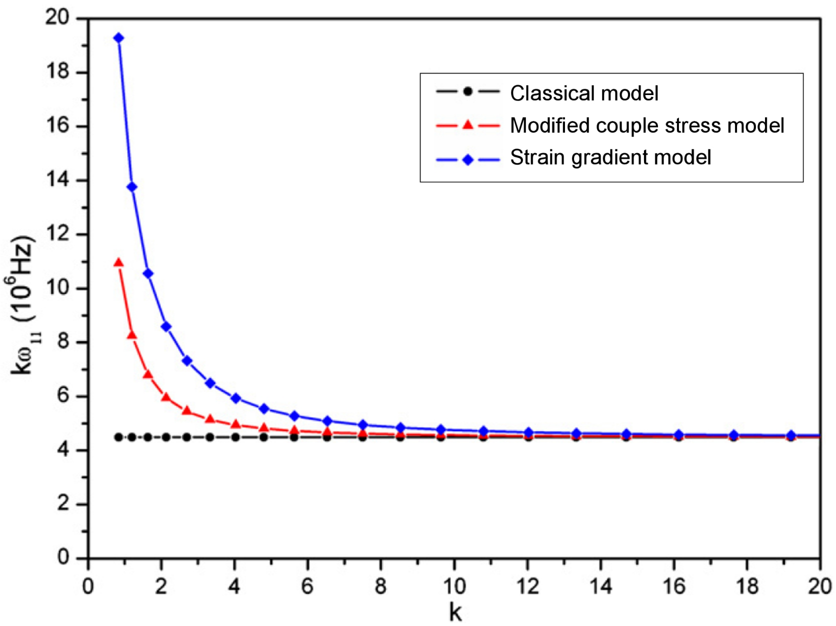


(b) Free vibration analysis

Fig. 6 Comparison between strain gradient model and couple stress model for microbeams [311]



(a) Bending analysis



(b) Free vibration analysis

Fig. 7 Comparison between strain gradient model and couple stress model for microplates [352]

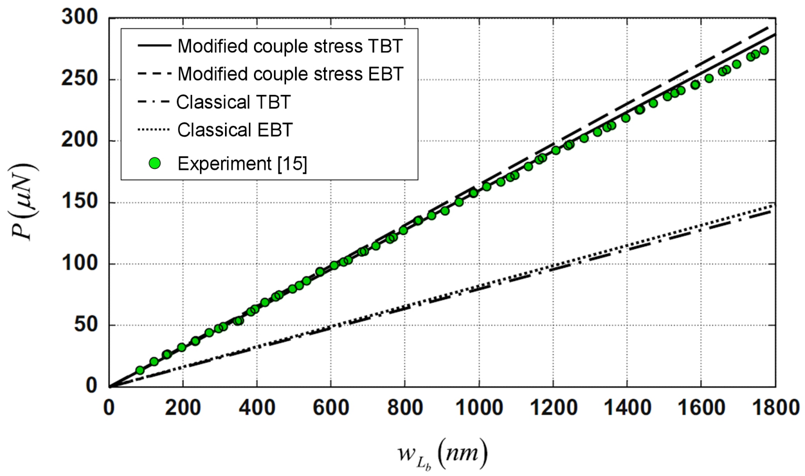


Fig. 8 Comparison of couple stress model with experimental result for cantilever microbeams [395]

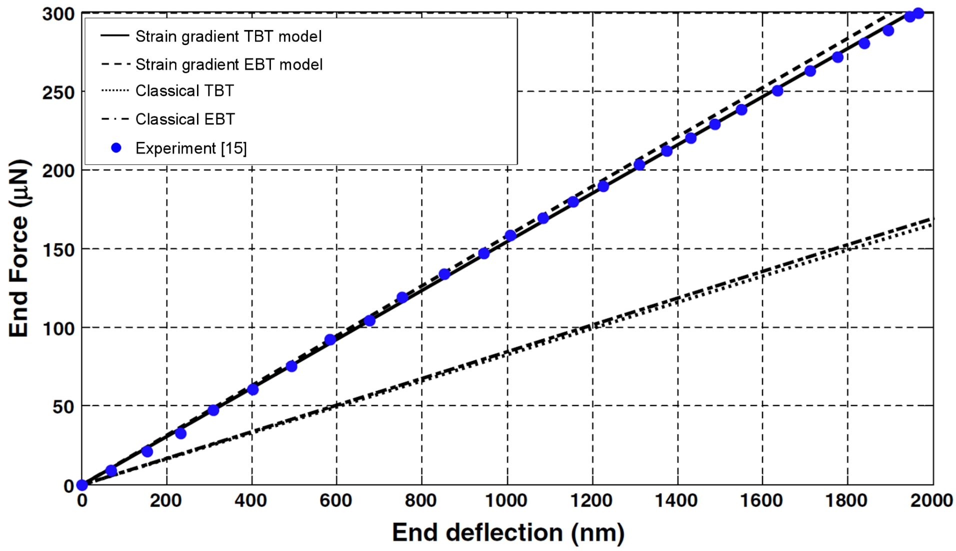


Fig. 9 Comparison of strain gradient model with experimental result for cantilever microbeams [401]

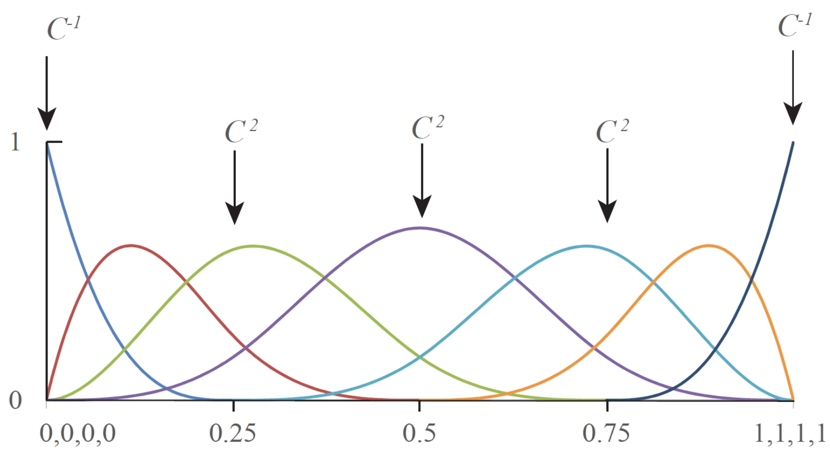


Fig. 10 NURBS basic functions

Table 1. Displacement field of HSDTs

Reference	Displacement field	Shape function	Unknown
Aydogdu [106] Karama et al. [107] Touratier [113] Soldatos [193]	$u_x(x,y,z,t)=u(x,y,t)-z\dfrac{\partial w}{\partial x}+f(z)\varphi_x(x,y,t)$ $u_y(x,y,z,t)=v(x,y,t)-z\dfrac{\partial w}{\partial y}+f(z)\varphi_y(x,y,t)$ $u_z(x,y,z,t)=w(x,y,t)$	$f(z)=z\alpha^{\frac{-2(z/h)^2}{\ln\alpha}}$, with $\alpha>0$ [106] $f(z)=ze^{-2(z/h)^2}$ [107] $f(z)=\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$ [113] $f(z)=h\sinh\left(\dfrac{z}{h}\right)-z\cosh\dfrac{1}{2}$ [193]	$u,v,w,\varphi_x,\varphi_y$
Shimpi [109] Thai and Choi [121] Thai and Vo [191] Darijani and Mohammadabadi [244] Darijani and Shahdadi [303]	$u_x(x,y,z,t)=u(x,y,t)-z\dfrac{\partial w_b}{\partial x}-f(z)\dfrac{\partial w_s}{\partial x}$ $u_y(x,y,z,t)=v(x,y,t)-z\dfrac{\partial w_b}{\partial y}-f(z)\dfrac{\partial w_s}{\partial y}$ $u_z(x,y,z,t)=w_b(x,y,t)+w_s(x,y,t)$	$f(z)=\left[\dfrac{5}{3}\left(\dfrac{z}{h}\right)^2-\dfrac{1}{4}\right]z$ [109] $f(z)=\dfrac{4z^3}{3h^2}$ [121] $f(z)=z-\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$ [121, 191] $f(z)=z-h\sinh\left(\dfrac{z}{h}\right)+z\cosh\dfrac{1}{2}$ [121] $f(z)=z-ze^{-2(z/h)^2}$ [121] $f(z)=\dfrac{z^5}{20h^4}-\dfrac{z^3}{24h^2}-\dfrac{63z}{64}+\alpha$ [244] $f(z)=z\left[-\dfrac{121}{16}+\dfrac{455}{4}\left(\dfrac{z}{h}\right)^2-483\left(\dfrac{z}{h}\right)^4+660\left(\dfrac{z}{h}\right)^6\right]$ [303]	u,v,w_b,w_s
Xiang et al. [343]	$u_x(x,y,z,t)=u(x,y,t)+z\varphi_x(x,y,t)-f(z)\left(\varphi_x+\dfrac{\partial w}{\partial x}\right)$ $u_y(x,y,z,t)=v(x,y,t)+z\varphi_y(x,y,t)-f(z)\left(\varphi_y+\dfrac{\partial w}{\partial y}\right)$ $u_z(x,y,z,t)=w(x,y,t)$	$f(z)=\dfrac{1}{n}\left(\dfrac{2}{h}\right)^{n-1}z^n$, with $n=3,5,7,9...$	$u,v,w,\varphi_x,\varphi_y$

Note: h is the thickness. For the beam model, the displacement u_y is equal to zero, and all non-zero generalised displacements are independent of the y coordinate.

Table 2. Displacement field of quasi-3D theories

References	Displacement field	Shape function	Unknown
Thai and Kim [115] Sobhy and Radwan [196] Thai et al. [250] Lei et al. [310] Trinh et al. [307]	$u_x(x,y,z,t)=u(x,y,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x}$ $u_y(x,y,z,t)=v(x,y,t)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y}$ $u_z(x,y,z,t)=w_b(x,y,t)+w_s(x,y,t)+g(z)w_z(x,y,t)$	$f(z)=z-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$ [115, 307] $f(z)=z-\frac{z\cosh\left(\frac{\pi}{2}\right)-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)}{\cosh\left(\frac{\pi}{2}\right)-1}$ [196] $f(z)=z-h\sinh\left(\frac{z}{h}\right)+z\cosh\frac{1}{2}$ [250] $f(z)=\frac{4z^3}{3h^2}$ [307, 310] $g(z)=1-f'(z)$	u,v,w_b,w_s,w_z
Reddy and Kim [308]	$u_x(x,y,z,t)=u(x,y,t)+z\theta_x+z^2\phi_x+z^3\psi_x$ $u_y(x,y,z,t)=u(x,y,t)+z\theta_y+z^2\phi_y+z^3\psi_y$ $u_z(x,y,z,t)=w(x,y,t)+z\theta_z+z^2\phi_z$		$u,v,w,\theta_x,\theta_y,\theta_z,$ $\phi_x,\phi_y,\phi_z,\psi_x,\psi_y$
Nguyen et al. [410, 417]	$u_x(x,y,z,t)=u(x,y,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x}$ $u_y(x,y,z,t)=v(x,y,t)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y}$ $u_z(x,y,z,t)=w_b(x,y,t)+g(z)w_s(x,y,t)$	$f(z)=\pi z\left(-1+\frac{9z^2}{5h^2}-\frac{28z^4}{25h^4}\right), g(z)=\frac{1}{8}f'(z)$ [410] $f(z)=z\left(8-\frac{10z^2}{3h^2}-\frac{6z^4}{5h^4}-\frac{8z^6}{7h^6}\right), g(z)=\frac{3}{20}f'(z)$ [417]	u,v,w_b,w_s

Note: h is the thickness. For the beam model, the displacement u_y is equal to zero, and all non-zero generalised displacements are independent of the y coordinate.