Review of nonlinear analysis and modelling of steel and composite structures

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Abstract

Structural steel frames exhibit significantly geometric and material nonlinearities which can be captured using the second-order inelastic analysis, also known as advanced analysis. Current specifications of most modern steel design codes, e.g., American code AISC360, European code EC3, Chinese code GB50017 and Australian code AS4100 permit the use of advanced analysis methods for the direct design of steel structures to avoid tedious member capacity checks. In the past three decades, a huge number of advanced analysis and modelling methods have been developed to predict the behaviour of steel and composite frames. This paper presents a comprehensive review of their developments, which focus on beam-column elements with close attention to the way to capture geometric and material nonlinearity effects. A brief outline of analysis methods and analysis tools for frames was presented in the initial part of the paper. This was followed by a discussion on the development of displacement-based, force-based and mixed beam elements with distributed plasticity and concentrated plasticity models. The modelling of frames subjected to fire and explosion was also discussed. Finally, a review of the beam-column models for composite structures including concrete-filled steel tubular (CFST) columns, composite beams and composite frames was presented.

Keywords: Advanced analysis; nonlinear analysis tool; steel frame; composite frame; beam-column model

1. Introduction

Steel and steel-concrete composite frames have been increasingly used in the high-rise buildings. Extensive research on the modelling of these structures has been carried out to develop beam elements for capturing geometric and material nonlinear effects. Geometric nonlinearities come from (i) the axial force acting through the displacement due to member curvature (P - δ effect) and member chord rotation (P - Δ effect), (ii) the axial shortening due to member bending (bowing effect), (iii) the out-of-straightness and out-of-plumbness (geometric imperfection effect) and (iv) the interaction between the axial, flexural, torsional and warping deformations. Material nonlinearities are associated with residual stresses and their inelastic behaviour. Geometric nonlinearity can be captured in the nonlinear analysis, whilst material nonlinearity is considered in the inelastic analysis.

Early work on the second-order analysis of framed structures started in the 1960s [1]. Based on the reference configuration used to describe the motion of element, there are three different formulations for handling geometric nonlinearities including updated Lagrangian (UL), total Lagrangian (TL) and co-rotational (CR) formulations. In TL formulation, the reference configuration is the initial undeformed state, whereas the UL formulation adopts the last known deformed state as the reference configuration. The CR formulation excludes the rigid-body motion, and thus the reference configuration constantly rotates and translates with the element in the rigid-body mode [2]. The stiffness matrix of a beam model can be formulated using either finite element method (FEM) or beam-column approach. The

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FEM is based on the energy method, whilst the beam-column approach is based on the beam-column theory. The merit of the FEM is its simplicity, whereas the advantage of the beam-column approach lies in its accuracy.

The inelastic analysis can be categorised into distributed plasticity type and concentrated plasticity type based on the degree of refinement used in modelling yielding. The distributed plasticity model allows yielding to form anywhere along the member length, whilst the concentrated plasticity model assumes that yielding is lumped at the member ends. The inelastic behaviour at a cross-section in both types can be defined using either stress-strain relationship or force-deformation relationship in terms of the stress resultants. The distributed plasticity model can provide the most accurate prediction of inelastic behaviours, but it is too computational cost. Whereas, the concentrated plasticity model is very computational efficiency, but its accuracy is less than that of the distributed plasticity model. Beam-column elements can be formulated using (i) the displacement method, (ii) the force method and (iii) the mixed method. The displacement method, which is the most widely used, is based on interpolating displacement fields, whilst the force method is based on interpolating internal forces. Meanwhile, the mixed method is just an extension of the force method where both force and displacement fields are separately interpolated.

A review of the geometric nonlinear analysis of framed structures was presented by Yang et al. [3]. Their review is limited to the papers published before 2002, and the works involving material nonlinearities of steel and composite frames were not considered. Liew [4] and Spacone and El-Tawil [5] reviewed the nonlinear analysis of composite structures including beams, frames and shear walls. However, their reviews are limited to the papers published before 2002 dealing with inelastic modelling of composite structures. Although significant works on advanced analysis and modelling of frames have been made in the last two decades, no review of recent advancements on advanced analysis and modelling of steel and composite frames has been reported in the open literature. Therefore, this paper aims to present a critical review of these works, which focus on beam/beam-column "line" elements with an emphasis on the way to capture both geometric and material nonlinearity effects. For the second-order analysis, this review covers all elements based on cubic, higher-order and stability functions. For the inelastic analysis, all inelastic models ranging from the simple elastic plastic to rigorous plastic-zone methods are reported. A detailed review of beam models for simulating the response of structures in fire and explosion is also presented. The last section covers the beam models for modelling steel-concrete composite structures.

2. Types of frame analysis

Common analysis methods for framed structures are summarized in Fig. 1 with their corresponding load-displacement behaviour. The linear elastic analysis is the simplest one which ignores both geometric and material nonlinearities and thus its load-displacement prediction is linear. The nonlinear elastic analysis accounts for geometric nonlinearity, but it ignores material nonlinearity. The elastic buckling limit calculated by an eigenvalue analysis represents an upper bound solution. In most practical designs, the $P-\delta$ effect is small and may be omitted in the frame analysis. However, in extreme cases, the amplification of moments and deflections due to P - δ effect can be as large as twenty percent of those due to the P - Δ effect [6]. Therefore, it should be considered in the analysis. The linear inelastic analysis accounts for material nonlinearities, but it ignores geometric nonlinearities. The plastic limit calculated by a plastic analysis represents an upper bound solution. The nonlinear inelastic analysis captures both geometric and material nonlinear effects. This is the only analysis method which can provide solutions close to real behaviour. Based on the nonlinear beam-column models, many nonlinear analysis programs were also developed to simulate the nonlinear inelastic behaviour of a whole framing structure. However, this review only limits to non-commercial packages involved in steel and composite frames which were developed for research purposes. A list of nonlinear analysis programs was summarised in Table 1 in a chronological order.

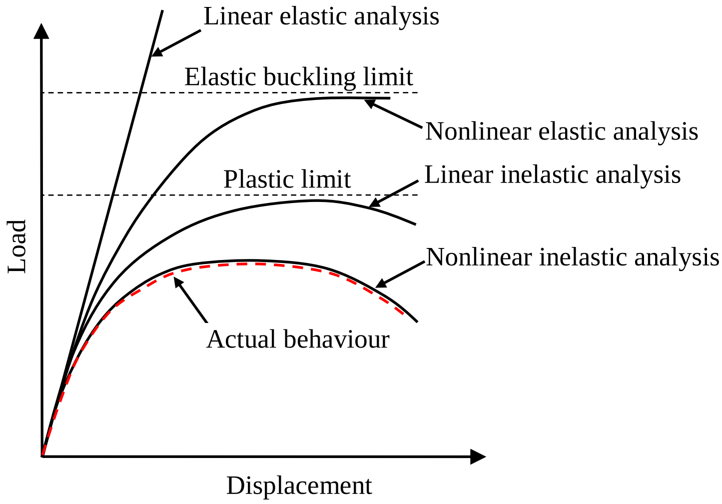


Fig. 1 Type of analysis [7]

DRAIN-2D (Dynamic Response Analysis of Inelastic 2-Dimensional Structures) is the first computer program deals with the second-order analysis of inelastic frames under both static and dynamic loading. It was developed by the University of California at Berkeley in 1973 based on the work of Kaanan [8]. The advantage of this program lies in its simplicity and efficiency. However, its capabilities are very limited. Therefore, it was updated to a more powerful version named DRAIN-2DX. The 3D version of DRAIN-2DX called DRAIN-3DX was also developed by Prakash [9] for 3D frames. Both DRAIN-2DX and DRAIN-3DX have been widely used in research for many years because they are not only simpler than other general-purpose nonlinear analysis programs, but also provide a number of important features for nonlinear seismic analysis which are not available by other programs.

INSTAF (INelastic STability Analysis of Frames) is a nonlinear inelastic analysis program for 2D steel frames with I-sections. The beam-column element implemented in this program was developed by El-Zanaty et al. [10] based on the fibre hinge concept in which each member is divided into three segments with the central segment being 80% of the member length. Therefore, it can effectively capture the inelastic effects due to residual stresses, strain hardening and gradual yield at the cross-section and over member. However, this program only limits to steel frames under static loads.

CEPAO (Calcul Elasto-Plastique Analyse-Optimisation) is a computer program for elasto-plastic analysis and optimisation of planar frames developed at the University of Liege in 1984 [11]. It was based on a combination of linear programming technique and FEM. The program was also extended to 3D steel frames incorporating the local buckling and strain hardening effects [12]. However, this program is not commonly used in the research community.

ADAPTIC is a nonlinear static and dynamic analysis program which was developed at Imperial College London based on Izzuddin's PhD study [13]. This is a pioneering computer program which employs adaptive mesh to save computational time in progressive collapse analysis of multi-storey buildings. It is a robust computational tool for the nonlinear analysis of inelastic buildings with slabs, shells and framed structures. ADAPTIC was initially developed for offshore jackets under static and dynamic loading and later has been extensively extended to various structural forms subjected to extreme loadings such as fire and blast.

PAAP (Practical Advanced Analysis Program) was developed at Sejong University based on Kim's PhD study [14]. It was initially developed for advanced analysis and design of 2D steel frames and then has been extensively developed by Thai and Kim [15, 16] for 3D structures with three nonlinear elements: beam-column element, truss element [17, 18] and cable element [19]. The most powerful feature of PAAP is its computational efficiency because of the use of a refined plastic hinge model (capture materially nonlinear effect) combined with stability functions (capture geometrically nonlinear effect).

OpenSees (Open System for Earthquake Engineering Simulation) is an extensible open-source platform developed at the University of California at Berkeley for simulating structures under earthquakes. It has been developed since 1998 by Pacific Earthquake Engineering Research (PEER) researchers with an aim to educate students and share new knowledge. Since then it has been extensively developed with over 160 element types and 220 material types mainly contributed from the research community.

FRAME3D is a computer program for the 3D nonlinear dynamic analysis of buildings developed by Krishnan [20] at California Institute of Technology. It has the capability of full 3D modelling steel buildings including steel frames, floor slabs, foundations and supports.

NEFCAD, which was developed by Chiorean [21] at Technical University of Cluj-Napoca, is a nonlinear inelastic analysis computer program for framed structures. This program included both plastic hinge and plastic zone elements as well as semi-rigid connections. Recently, NEFCAD has been extended to composite frames [22].

Table 1. Summary of nonlinear analysis tools for steel and composite frames

Name	Year	Developed by	Comments
DRAIN-2D/2DX/3DX	1973	University of California, Berkeley	Steel frames under static and dynamic loading
INSTAF	1980	University of Alberta	Steel frames under static loading only
CEPAO	1984	University of Liege	Linear programming technique
ADAPTIC	1990	Imperial College London	Adaptive mesh to save computational time
PAAP	1996	Sejong University	Using stability functions to capture P - δ effect
OpenSees	1998	University of California, Berkeley	Force-based beam elements
FRAME3D	2003	California Institute of Technology	Full 3D modelling of steel buildings
NEFCAD	2005	Technical University of Cluj-Napoca	Steel and composite frames

3. Geometrically nonlinear analysis

3.1. Displacement-based element

3.1.1. Cubic element based on Lagrangian approach

Bathe and Bolourchi [23] presented both TL and UL formulations of a cubic element for the large displacement analysis of 3D frames. They concluded that the UL formulation is more computationally effective than the TL one [23]. A detailed comparison between them was also carried out by Narayanan and Krishnamoorthy [24]. Remseth [25] improved the accuracy of cubic element by using higher-order polynomials for the axial displacement. Cichon [26] presented a TL cubic element with local-global degree-of-freedom (DOF) for the large displacement inelastic analysis of 2D frames. Yang and Saigal [27] incorporated a fibre model into a cubic element developed by Yang [28] for the nonlinear inelastic analysis of beams under static and dynamic loads. Espion [29] developed a three-node UL cubic element with nine DOFs for the nonlinear inelastic analysis of 2D frames. Meek and Tan [30, 31] developed a cubic element accounting for bowing effect. This element was used by Meek and Loganathan [32, 33] to examine the post-buckling behaviour of 3D frames. Chajes and Churchill [34] illustrated the implementation of a cubic element for the nonlinear analysis of frames using three procedures including linear/nonlinear incremental method and direct method. A comparison of three kinematic formulations for updating geometry including total and increment secant stiffness method and joint orientation method was carried out by Chan [35] for the large deflection analysis of 3D frames. Numerical results indicated that the total secant stiffness method is the simplest approach to program, but it is unable to capture the geometric nonlinear effect of frames due to large rotations. The increment secant stiffness method is good at convergence, but it is less accurate than the joint orientation method. The joint orientation method is reliable and accurate but its implementation is more complicated than the two other approaches [35]. The increment secant stiffness method was used by Ho and Chan [36] for the nonlinear analysis of inelastic semi-rigid steel frames. Chan [37] also adopted the incremental secant stiffness method to develop a robust numerical procedure for the large deflection analysis of 3D frames under dynamic loading.

Yang and McGuire [38] proposed a procedure including the effect of warping restraint on the nonlinear analysis of 3D frames. Yang and McGuire [39] derived the stiffness matrix of a UL cubic element accounting for warping effects of members with doubly symmetric thin-walled cross-sections. The stiffness matrix formulations for asymmetric thin-walled cross-sections were derived by Kitipornchai and Chan [40] without the warping effect and by Chan and Kitipornchai [41] with warping effect. Chan [42] presented a numerical strategy for nonlinear problems using the minimum residual displacement method. Al-Bermani and Kitipornchai [43] improved the previous work in [40, 41] by introducing an additional matrix to account for the higher-order nonlinear terms in the kinematics of the element. Therefore, accurate prediction can be obtained by using one element per member. The model was then employed by Al-Bermani and Kitipornchai [44], Al-Bermani et al. [45] and Zhu et al. [46] to investigate the nonlinear behaviour of semi-rigid frames.

To improve the accuracy of the UL cubic element in predicting the P - δ effect, So and Chan [47] proposed a fourth-order element by adding a translational DOFs to the middle of the element. Verification studies indicated that a single fourth-order element is sufficiently accurate for the buckling analysis of frames. Yang and Leu [48, 49] also improved the accuracy of the cubic element by deriving higher-order stiffness matrices and used them in the force recovery procedure. They concluded that the corrector phase is more important than the predictor phase in governing the accuracy of an incremental-iterative scheme. The use of better predictors only accelerates the convergence speed. Yang et al. [50] also confirmed that for moderately nonlinear problems, the use of linear stiffness matrix is sufficient for most practical purposes. However, for highly nonlinear problems which exhibit winding loops in the post-buckling response, both geometric and linear stiffness matrices should be used to ensure that iterations do not diverge or converge to incorrect directions. Yang et al. [51] derived the elastic and geometric stiffness matrices of a cubic element based on the small-deformation theory and a UL formulation. Gu and Chan [52] developed a new tangent stiffness matrix of a refined cubic element which allows for higher-order effects, and thus it is capable of accurately predicting torsional and flexural-torsional buckling phenomena. Recently, Bai et al. [53] developed a cubic element incorporating the initial imperfection for the practical design of tapered I-section columns.

3.1.2. Cubic element based on co-rotational approach

CR formulations were first proposed by Argyris et al. [54] by separating rigid body motions from total deformations. They were later applied to frames by Jennings [55], Powell [56], Oran [57, 58], Belytschko and Hsieh [59], Belytschko and Glaum [60] and Izzuddin and Elnashai [61] using different terminologies such as natural approach [62], Eulerian approach [57, 58, 61] and convected coordinate [59]. The CR terminology was first introduced by Belytschko and Glaum [60] to refer to the motion of a local coordinate system attached to the element. Teh and Clarke [2] indicated that the UL cubic element can be as accurate as the CR one if it accounts for the deformed configuration at the last known state. Belytschko and Glaum [60] developed a higher-order CR formulation which accounts for moderate rotations and initial curvature of beams. They concluded that the higher-order CR formulation converges to the exact solution more rapidly than the lower order CR formulation when the mesh is insufficiently refined. Izzuddin [63] clarified confused conceptual issues involving the second-order analysis of 3D frames including the nature of end moments and the symmetry of the tangent stiffness matrix.

To handle problems involving large rotations, Rankin and Brogan [64] proposed a CR procedure which enables standard element formulations to be embedded. Hsiao and Hou [65] proposed a simple procedure for the large displacement analysis of 2D frames with large rotations. This procedure was also extended by Hsiao et al. [66] to include material nonlinearities. In order to deal with the large rotation of 3D frames, Hsiao et al. [67] proposed a CR procedure by simply applying the transformation matrix to linear tangent stiffness matrix. However, the variation of the transformation matrix was not properly captured by this procedure. Crisfield [68] first presented a variationally consistent CR formulation for 3D beams element, in which both internal force vector and tangent stiffness matrix were derived in a consistent manner. The benefit of the variationally consistent element lies in its fast convergence. Nguyen [69] developed a CR element in the context of Timoshenko beam theory, and thus accounting for the shear deformation effect in deep beams.

Hsiao [70] formulated a CR element in the TL framework for 3D beams with large rotations. All coupling terms including bending, twisting and stretching deformations which are essential for the lateral-torsional buckling of 3D beam elements were considered by using the fully geometrically nonlinear theory. This element was also employed by Hsiao et al. [71] to study the linear buckling of 3D frames. Hsiao and Lin [72] improved the work in [70] by including the effects of the third-order nonlinear terms of the twist rate which are significant for a narrow rectangular cantilever beam under twist. It should be noted that those third-order terms were included in an earlier formulation developed by Attard [73].

The extension of CR formulations to thin-walled members was carried out by Izzuddin and Smith [74, 75]. The CR formulation in [70] was extended to include the warping effect of thin-walled beams with mono-symmetric cross-sections by Hsiao and Lin [76], doubly symmetric cross-sections by Lin and Hsiao [77] and generic cross-sections by Chen et al. [78]. Battini and Pacoste [79] developed both Euler-Bernoulli (EB) and Timoshenko elements for thin-walled frames based on the CR framework proposed by Pacoste and Eriksson [80]. The work in [79] was extended by Battini and Pacoste [81] and Alsafadie et al. [82] to include material nonlinearities. Alsafadie et al. [83] improved their previous work in [82] by including higher-order terms of bending curvature in the local formulation, and thus requiring a less number of elements in the modelling of structures. Battini [84] presented a CR element without rotational DOFs. In this element, the nodal rotations were extrapolated using the displacements of a set of four nodes including the two nodes of the element and two additional internal nodes. Le et al. [85] extended their previous consistent CR formulations in [86] to thin-walled beams with arbitrary cross-sections.

The CR formulation was also used for the frames under dynamic loads. Belytschko and Hsieh [59] presented a CR element for advanced analysis of beams with small rotations. The formulation was extended by Belytschko et al. [87] to 3D frames. Crisfield et al. [88] developed a 3D CR element for the dynamic analysis of spatial beams in the framework suggested by Rankin and Brogan [64] for large rotations. The inertia terms were derived based on Timoshenko mass matrix. Hsiao and Jang [89] extended the static work in [65] to the dynamic analysis of 2D frames [89]. However, the inertia nodal force vectors used in [89] were derived from the linear beam theory which is unable to account for complete inertia effects. These effects were included in a consistent CR element proposed by Hsiao et al. [90] with the inertia nodal forces derived from the fully nonlinear beam theory. Hsiao and Yang [91] further extended the work in [90] to include initial curvature effects for the curved beams. Hsiao et al. [92] presented a consistent CR element for the nonlinear dynamic analysis of 3D beams by extending the CR formulation in [70]. Le et al. [93] derived a new mass matrix for CR element using cubic interpolation functions. The new mass matrix formulation is more accurate than the conventional lumped mass matrix, but it requires more computational time. The extension of the works in [93] to 3D beams was also carried out by Le et al. [86, 94]. Chhang et al. [95, 96] developed a CR element for the nonlinear time-history analysis of planar frames using an energy-momentum integration scheme, whilst Alhasawi et al. [97] proposed a CR beam element generalised elasto-plastic hinges for the nonlinear inelastic cyclic analysis of 2D frames. This element was recently improved by Heng et al. [98] to include a contact element for the nonlinear dynamic analysis of planar frames under impact.

3.1.3. Beam-column element based on stability functions

As stated by Teh [99], the CR cubic element is sufficiently accurate for the second-order analysis of practical steel frames. However, this statement is only true for the nonlinear problem governed by the P - Δ effect [100]. For the nonlinear problem controlled by the P - δ effect, more CR cubic elements per member are still required to convert the P - δ effect into the P - Δ effect. In an attempt to capture accurately the P - δ effect using the least element per member, the beam-column approach was introduced using stability functions derived from the differential equations of a beam-column member under axial compression. The derivation of stability functions can be found in [101-103] for 2D members and in [104] for 3D ones. In beam-column method, the tangent stiffness matrix of the element is derived from the basic member force-deformation relationship, i.e., without rigid-body motions. The disadvantage of this approach is that it cannot predict lateral-torsional buckling since the coupling term between torsional and flexural displacements is lost when a direct extension from a 2D element to a 3D element is made.

The stiffness matrices were derived by Oran for a 2D element [57] and a 3D element [58] based on stability functions and bowing functions. The 2D element in [57] was validated by Oran and Kassimali [105] for 2D frames under static and dynamic loads. Kassimali [106] extended the 2D element in [57] to include material nonlinearities. The stiffness formulation of the 3D element in [58] was improved by Kassimali and Abbasnia [107] to include the effects of axial force on torsional stiffness and some coupling stiffness terms and Chan and Gu [108] to include initial geometric imperfections. Abbasnia and Kassimali [109] extended their work in [107] to account for material nonlinearities. Kam [110] proposed the tangent stiffness matrix of a beam element which accounts for the P - δ effect, bowing effect and the spread of plasticity. Dumir et al. [111] derived the stiffness matrix of a beam element under distributed loads when the bowing effect was neglected. Namini et al. [112] presented a consistent mass matrix of a 3D beam-column element accounting for axial-flexural coupling effect. Lui and Chen [113-115] and Chen and Lui [116] proposed a second-order inelastic analysis method for semi-rigid frames based on the beam-column method. The geometric nonlinearity is captured using the stability functions and UL formulation, whilst the material nonlinearity is included using an elastic-plastic model. Balling and Lyon [117] developed a CR element for the second-order analysis of planar frames using the stability functions.

Although the stability functions have been widely used in the stability analysis of steel frames because of its accuracy in predicting the P – δ effect, they may become numerically unstable when the axial force is small. To overcome this limitation and avoid using different expressions for compressive and tensile axial forces, Lui [118] approximated the stability functions using a Taylor series expansion. The stability functions were also approximated by Goto and Chen [119] using a power series approach, and recently by Doan-Ngoc et al. [120] using a seventh-order polynomial.

3.1.4. Element based higher-order polynomials

Higher-order beam elements based on the higher-order polynomial interpolation functions were proposed to improve the accuracy of cubic elements and eliminate the numerical instability of stability function-based elements when the axial load is small. Based on a fifth-order polynomial, Chan and Zhou [121] proposed a pointwise equilibrating polynomial (PEP) element capable of accurately predicting the P - δ effect in most practical cases. A comparison between the approximate fifth-order form [121] and the exact form of stability functions shown in Fig. 2 indicates that the fifth-order form approximates well with the stability functions when the compressive axial force P is less than 3.0 times the Euler's buckling load P_e. If P is greater than 3.0 times P_e, the error for the fifth-order form increases rapidly as the compressive axial force P increases. The PEP element was improved by Chan and Zhou [122] and Zhou and Chan [123-125] to include geometric imperfections [122], semi-rigid connections [123] and distributed loads along the member [124, 125]. The PEP element was extended by Zhou and Chan [126] to allow one plastic-hinge to form along the member length, and thus it can predict the large deflection inelastic response of steel frames by using one element per member in the modelling. Chan and Zhou [127] also extended their previous work [126] to permit the formation of three plastic-hinges located at the two ends and at an arbitrary location along the length. Recently, Tang et al. [128] rederived the PEP element in the framework of the Timoshenko beam theory to capture the shear deformation effect in deep beams.

Izzuddin [129] presented a fourth-order element in the CR framework for frames under thermal effects and distributed loads. The element was also implemented in ADAPTIC program developed by himself. Iu and Bradford [130] also developed a fourth-order element for the nonlinear analysis of 3D frames, but it was based on a UL formulation. Verification studies indicated that the element can capture the geometric nonlinear effects due to large deflection, snap-through buckling and bowing effect by using one element per member. Liew et al. [131] developed a UL element which is capable of accurately capturing the P - δ effect, bowing effect, axial-torsional and lateral-torsional buckling effect and initial geometric imperfections. Their element employs the merits of stability functions, Hermitian functions and PEP functions.

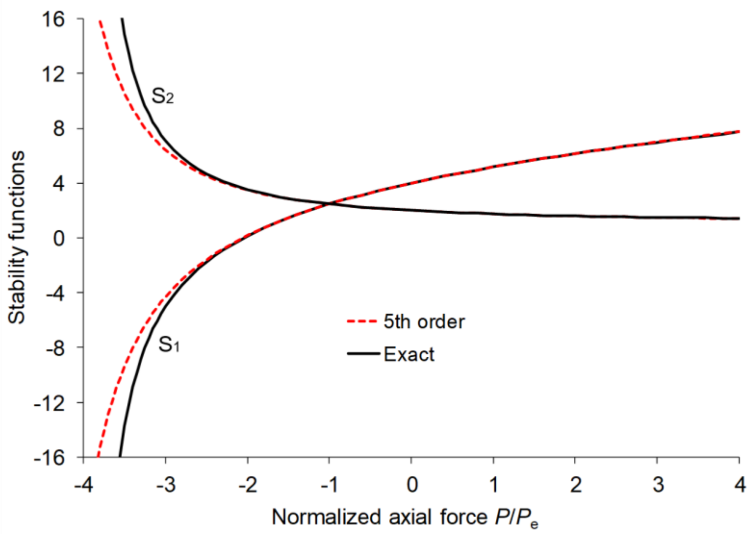


Fig. 2 Comparison between the 5th-order and exact forms of stability functions S_1 and S_2

3.2. Force-based element

The force-based element is formulated in a natural system, i.e., without rigid-body motions, using force interpolation functions which satisfy the equilibrium conditions of the stress resultants along the element. This element is especially beneficial for simulating material nonlinearities since the stress resultants at the section and member levels are always in equilibrium whatever material nonlinearity takes place. However, the force-based element is not widely used due to the difficulty in implementing the element state determination in an existing FEM usually based on the displacement-based framework. This difficulty was overcome in the element state determination proposed by Ciampi and Carlesimo [132] in which the element stiffness matrix and resisting forces were determined by an iterative process that equilibrium and compatibility conditions are always satisfied in a strict sense during the iteration. To eliminate the iterative process and further expand the benefits of the force-based element, Neuenhofer and Filippou [133] proposed an element state determination in which both residual displacements and unbalanced section forces were used to calculate the member forces.

Neuenhofer and Filippou [134] extended the force-based element proposed in [133] to include the P - δ effect by using the curvature-based displacement interpolation (CBDI) procedure, which adopts Lagrangian polynomials to approximate the displacement field. This procedure was extended by Jafari et al. [135] to account for the shear deformation effect in deep beams using a curvature-shear-based displacement interpolation (CSBDI) procedure. Based on the force formulation and higher-order beam theory, Correia et al. [136] proposed a higher-order element accounting for the flexural-shear-torsional interaction and the interaction between 3D shear and normal stresses. The application of this element was also illustrated by Almeida et al. [137].

3.3. Mixed element

Alemdar [138] developed both force-based and mixed elements for the second-order analysis of planar and 3D inelastic frames based on a CR formulation. Alemdar and White [139] compared the accuracy and efficiency of the displacement-based, force-based and mixed element in predicting the nonlinear behaviour of frames. It was confirmed that the mixed element gave the best results, and both force-based and mixed elements were more efficient than the displacement-based one in capturing the inelastic behaviour. To include shear deformation effects, Taylor et al. [140] developed 2D Timoshenko beam element based on three-field variational principle. The extension of this element to 3D was carried out by Saritas and Soydas [141].

The mixed approach was also applied to frames with thin-walled cross-sections. For example, Nukala and White [142, 143] proposed a mixed element for the second-order analysis of 3D inelastic steel frames with thin-walled cross-sections. This element accounts for finite rotations, warping effect and geometric imperfections as well as residual stresses. Since the stress resultants at the section level are interpolated from equilibrium equations, the accuracy of this element is significantly improved in highly nonlinear problems. Alsafadie et al. [144, 145] developed a 3D mixed element for the nonlinear analysis of beams with generic thin-walled cross-sections based on the EB theory [144] and Timoshenko theory [145]. The CR framework developed by Battini and Pacoste [79] was used to capture the geometric nonlinear effects due to large rotations and displacements. The bending-twisting-stretching coupling was also included. Alsafadie et al. [146] presented eight elements including four displacement-based elements and four mixed elements in the CR framework. These elements are suitable for the large deflection inelastic analysis of frames with generic thin-walled cross-sections. The similarity and discrepancy between different elements were highlighted.

4. Inelastic analysis

4.1. Distributed plasticity analysis

Typical names for this model can be found in the literature such as elasto-plastic analysis, plastic-zone analysis, spread of plasticity analysis and distributed inelasticity analysis. In the distributed plasticity analysis, the gradual yielding across the cross-section can be modelled using two different approaches. In the first approach, the gradual yielding is simulated using moment-thrust-curvature relations. The shortcoming of this method is that the moment-thrust-curvature data are only available for a certain cross-sectional geometry and stress-strain characteristics. One of the earliest studies using this approach was by Chu and Pabarcius [147] and Moses [148]. In the second method, the gradual yielding is captured through the use of a fibre model with the cross-section being subdivided into many fibres as illustrated in Fig. 3. The state of strain and stress of each fibre and its yield criteria are explicitly traced during the analysis, and thus capturing the gradual yield across the cross-section. This approach is recognised as the most accurate method since it can explicitly account for the effects of residual stresses and material strain hardening. Although this method is applicable to cross-sectional shapes of arbitrary geometry, it is too computationally intensive as a very refined discretization of the cross-section is required. Therefore, it is used for the research purpose to verify with plastic-hinge methods. One of the earliest studies using the fibre method was by Alvarez and Birnstiel [149]. It is worth mentioning that, in both moment-thrust-curvature and fibre methods, the element needs to be partitioned into a number of cross-sections along the member length as shown in Fig. 3 thus the gradual yielding along the member can be captured.

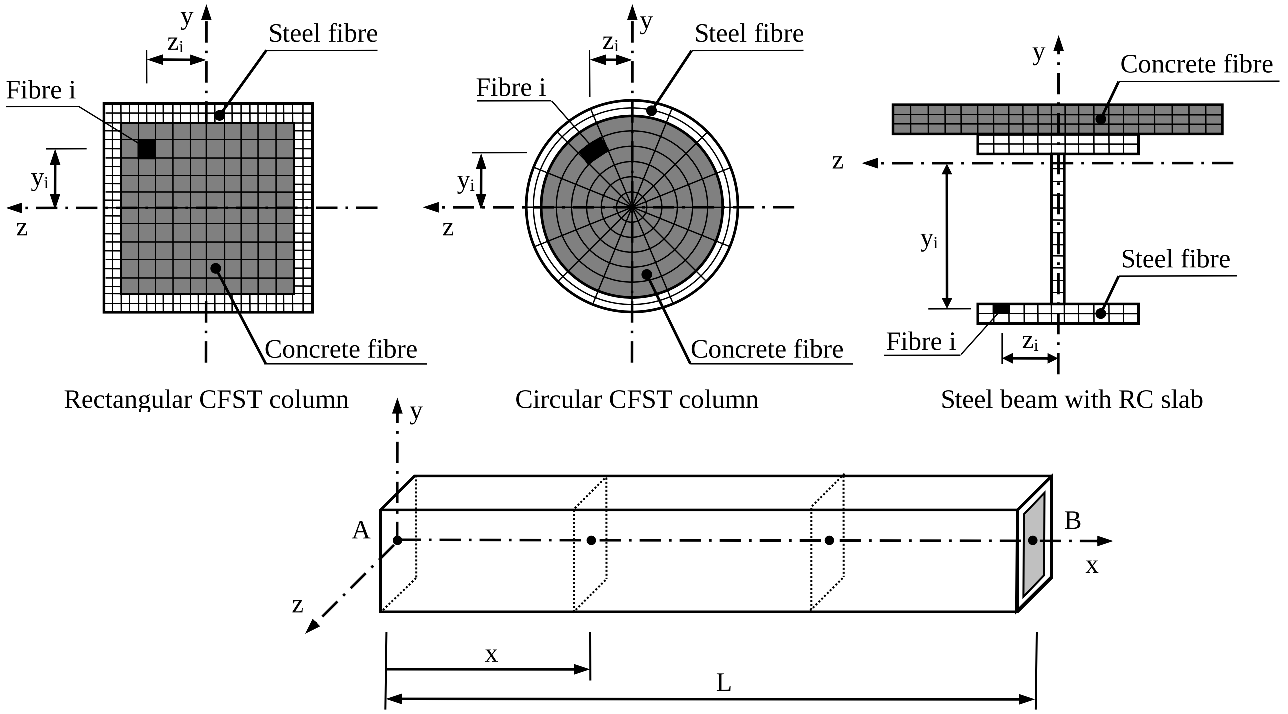


Fig. 3 Concept of fibre model for steel-concrete composite members

4.1.1. Displacement-based element

Meek and Lin [150] investigated the inelastic behaviour of thin-walled beam-columns using a fibre model with von Mises yield criterion. The effects of residual stresses and geometric imperfections were considered in the analysis. Verification studies indicated that their model based on both normal and shear stresses is more accurate than the fibre model based on the uniaxial stress when compared with the experimental results. Izzuddin and Smith [75] included the fibre model to a CR cubic element for the large deflection inelastic analysis of 3D thin-walled frames. They concluded that for lateral-torsional buckling problems, the fibre model based on uniaxial stress-strain relationships gives an excellent prediction of the inelastic buckling loads, whilst the use of biaxial stress-strain relationships is only needed to determine the post-buckling behaviour at large displacement [74]. Teh and Clarke [151] also presented a CR fibre element 3D inelastic steel frames with closed and open thin-walled cross-sections. Verification results indicated that the CR fibre element can accurately capture the nonlinear inelastic response of frames with a coarser mesh if a proper force recovery procedure was used. Jiang et al. [152] presented a UL fibre element for the plastic-zone analysis of 3D steel frames. A mixed element technique was also proposed to reduce computational time of the plastic-zone analysis for large-scale frames. In this technique, the member which is remained in elastic behaviour is modelled as one elastic element. If the severe yielding in the critical member is detected, it is divided into several plastic-zone elements along the member length to capture the gradual yielding. It is found that the computational efficiency of the mixed element is improved by more than 10 times compared with the plastic-zone element when analysing a 20-storey 3D steel frame.

Another efficient method to trace the yielding of fibres is to use uniaxial stress-strain relations. El-Zanaty and Murray [153] used a trilinear stress-strain relationship for the nonlinear analysis of 2D inelastic steel frames. The analysis accounts for all sources of material nonlinearities including strain hardening, residual stresses and gradual yield over the cross-section and along the length. The element was implemented in an inelastic stability analysis of frames (INSTAF) program for the nonlinear analysis of 2D inelastic steel frames. Chan and Kitipornchai [154] included a fibre model in their elastic beam element [41] to study the inelastic post-buckling behaviour of tubular struts. Kitipornchai et al. [155] extended the work in [154] to hollow section members. The effects of residual stress, geometric imperfections and strain-softening were taken into account in their study. Chan [156] proposed an efficient incremental-iterative procedure for the inelastic post-buckling analysis of tubular members using a fibre model and the minimum residual displacement method. Chan et al. [157] extended the work in [156] to include the local buckling in square hollow cross-sections by modifying the stress-strain relationship of materials. Meek and Loganathan [158] extended their previous works [32, 33] to account for material nonlinearities. Izzuddin and Elnashai [159] adopted the fibre model for the nonlinear analysis of inelastic frames based on the concept of automatic mesh refinement (adaptive mesh method). The advantage of this method is to save computational time since the discretisation process on the cross-section is performed only where required within the structure and when necessary during analysis. The plastic-zone analysis of 2D inelastic steel frames was presented by Clarke [160] using a fibre cubic element. Nguyen et al. [161] extended the plastic-zone method to 3D inelastic steel frames under dynamic loading. Foley and Vinnakota [162-164] developed a fibre cubic element for the nonlinear inelastic analysis of 2D steel semi-rigid frames. The element stiffness was modified based on a static condensation procedure to include the semi-rigid behaviour of beam-to-column connections. Thai and Kim [165] proposed a fibre element which can accurately predict both geometric and material nonlinearities by using only one element per member in the modelling. The geometric nonlinearity was captured using the stability functions, whilst the material nonlinearity was taken into account using the fibre model. Thai and Kim [166, 167] also extended the capability of their element to steel frames under earthquake [166] and CFST structures [167], whilst Nguyen and Kim [168] improved the element to account for the lateral-torsional buckling effect of 3D frames.

In addition to the fibre model, moment-thrust-curvature relations were also used in the distributed plasticity analysis to model the inelastic behaviour of frames. For example, Li and Lui [169] presented a simplified plastic-zone method for 2D steel frames. In this method, the gradual yielding over the cross-section was captured using an effective cross-sectional stiffness derived from moment-thrust-curvature relationships. The gradual yielding along the member was captured using the effective member stiffness derived from the trapezoidal integration rule. Barsan and Chiorean [170] presented the computer program NEFCAD for large displacement inelastic analysis of 2D steel semi-rigid frames. In this analysis, the member was divided into a number of elements, and the gradual yielding of each cross-section was accounted for using the Ramberg-Osgood moment-curvature relationship. Similar work was presented by Chiorean and Barsan [171], but it was based on the integration points (see Fig. 4) instead of discrete elements as in [170] to simulate the gradual yielding along the member length. Therefore, only one element per member is needed in the modelling. Chiorean extended the work in [171] to include semi-rigid connections. A force-strain curve was also proposed to account for the residual stresses.

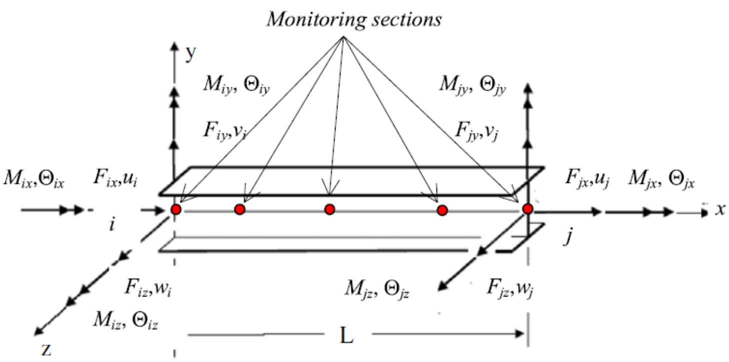


Fig. 4 Displacement-based fibre element [171]

4.1.2. Force-based element

The force-based fibre element was proposed by Spacone et al. [172, 173] for the inelastic analysis of reinforced concrete (RC) frames. The effect of the number of integration points at the monitored sections and the number of integration points over cross-sections were investigated by Kostic and Filippou [174]. Valipour and Foster [175] presented a novel force-based element for the inelastic analysis of RC frames based on a secant stiffness approach. In this element, the conventional fibre model was replaced by a numerical integration scheme to improve the accuracy and computational efficiency. Valipour and Foster [176] extended their previous work in [175] to include geometric nonlinearities by using the Simpson integration scheme with a parabolic interpolation of the curvature. This approach is more straightforward and less computational cost than the CBDI procedure proposed in [134]. To include the shear deformation effect, Papachristidis et al. [177] adopted the Timoshenko beam theory to formulate a force-based fibre element for the inelastic analysis of 3D steel frames. Chiorean [178] and Chiorean and Marchis [179] developed new force-based elements for steel frames that are capable of capturing geometric and material nonlinear effects and geometric imperfections by using only one element per member.

De Souza [180] presented 2D and 3D fibre elements for the large deflection analysis of inelastic frames. In these elements, the geometric nonlinearities due to large rotations and displacements are accounted for via the transformation of the force-deformation relationship of the basic element in the context of the CR formulation, whilst the geometric nonlinearity due to P - δ is captured using CBDI procedure. Material nonlinearity is included using a fibre model. Therefore, only one element per member is needed for predicting the nonlinear inelastic responses of almost practical problems. This force-based element was also implemented in OpenSees software. This element was also employed by Uriz et al. [181] and Thai and Kim [182] to predict the inelastic buckling of steel braces [181] and the nonlinear behaviour of semi-rigid inelastic steel frames [182]. Rezaiee-Pajand and Gharaei-Moghaddam [183, 184] extended De Souza's element to account for the shear deformation effect in Timoshenko beams, whilst Du et al. [185] included initial imperfections in this element. Therefore, it can be used for the practical design of steel structures. Recently, Du et al. [186-188] also extended their previous work in [185] to account for (i) shear deformation effect of deep beams [187], (ii) semi-rigid connections and rigid end zone for gusset plate connections [186] and (iii) local buckling of CFST columns with noncompact and slender sections [188].

4.1.3. Mixed element

Spacone et al. [189] developed a mixed element for the inelastic analysis of frames based on a two-field mixed formulation. Lee and Filippou [190] enhanced the mixed formulation of an EB element with an explicit stress interpolation function over the cross-section. Gkimousis and Koumousis [191] presented a mixed fibre element for the analysis of inelastic steel frames under cyclic loading. A new hysteretic uniaxial steel model was proposed to capture the kinematic hardening and the Bauschinger effects. Saritas and Filippou [192] extended their previous works in [140] to account for inelastic coupling of bending moment, axial force and shear force.

4.2. Concentrated plasticity analysis

In the concentrated plasticity model, the formation of the plastic-hinges is assumed to be at the two ends of the element. This model is widely used for advanced analysis and practical design of steel frames due to its simplicity and computational efficiency. In general, the concentrated plasticity models can be classified into elastic plastic-hinge and modified plastic-hinge. Early work on the plastic-hinge analysis was carried out by Nigam [193].

4.2.1. Elastic plastic-hinge method

This is the simplest inelastic analysis method. It accounts for inelasticity but ignores the spread of plasticity at cross-sections and residual stresses. The development of the plastic-hinges at two ends of the element is governed by the yield surface represented by the stress resultants. If the member force at a cross-section reaches the yield surface, the cross-section sharply changes from an elastic state to fully plastic state to form the plastic-hinge. An excellent work on the application of the elastic plastic-hinge method in a displacement-based element to predict the nonlinear behaviour of inelastic frames was carried out by Orbison [194]. Based on the plastic-hinge model and UL formulation, Hsieh and Deierlein [195] developed a beam model for the nonlinear analysis of 3D inelastic semi-rigid steel frames. The elastic plastic-hinge concept was also used by Haldar and Nee [196] for the nonlinear analysis of 2D inelastic frames. Nee and

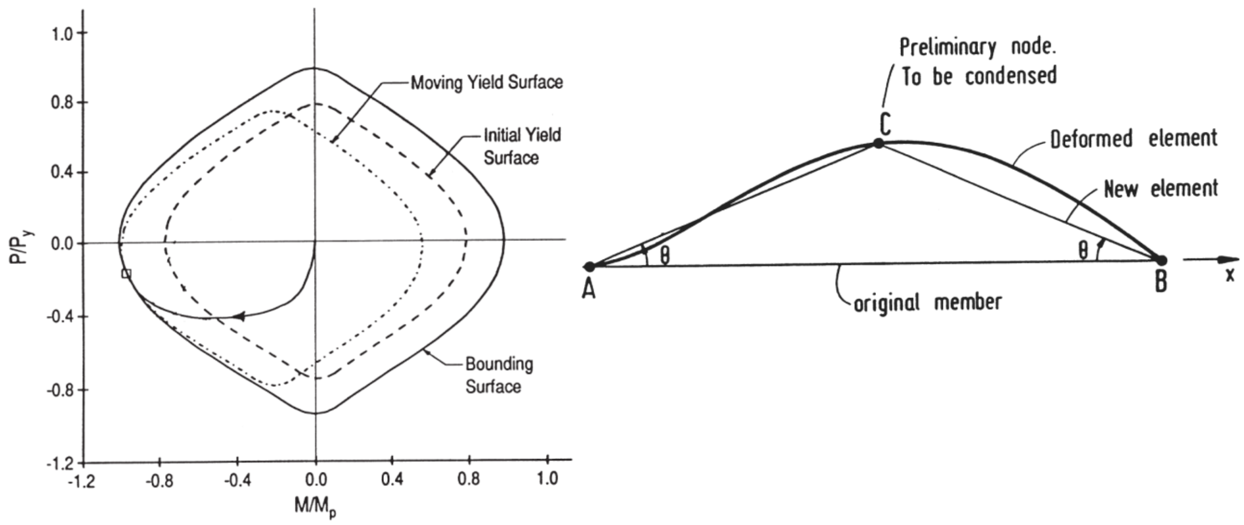
Haldar [197] extended the work of Haldar and Nee [196] to 3D frames. Chandra et al. [198] included an elastic plastic-hinge model in their previous work on the nonlinear behaviour of steel frames [199]. A yield surface was also derived for an I-shaped section. Orbison et al. [200] developed stress-resultant yield surfaces for steel cross-sections. Izzuddin and Elnashai [201] proposed a plastic hinge approach for space frames based on the concept of adaptive mesh refinement. In this approach, a plastic hinge element is subdivided into two elements if the inelastic behaviour occurs during analysis. Therefore, it can enhance the efficiency of the conventional plastic hinge method. Hoang and Nguyen [202, 203] modified the plastic-hinge model to include the strain hardening effects [202] and the local buckling check according to Eurocode 3 [203]. Their work was implemented in the computer software CEPAO.

4.2.2. Modified plastic-hinge methods

The modification of the elastic plastic-hinge approach aims to smooth the sharp plasticity. The modified plastic-hinge methods are comparable to the plastic-hinge method in simplicity, but their accuracy is comparable to that of the plastic-zone method. A huge number of modified plastic-hinge models have been proposed and they can be classified into four categories as follows.

4.2.2.1. Two-surface plastic-hinge model

The two-surface plastic-hinge model accounts for the gradual yield and hardening effects using the bounding surface concept and theory of plasticity. In this model, the gradual yielding over the cross-section is accounted for using two interaction surfaces. The inner surface which is identified as the initial yield surface bounds the elastic region, whilst the outer surface which is referred as the bounding surface represents the full yielding state. The evolution of the bounding surface for I-shaped sections under proportional and nonproportional cyclic loading was studied by Hajjar [204].



(a) Axial force and moment interaction surface (b) Element subdivision due to yielding at mid-span

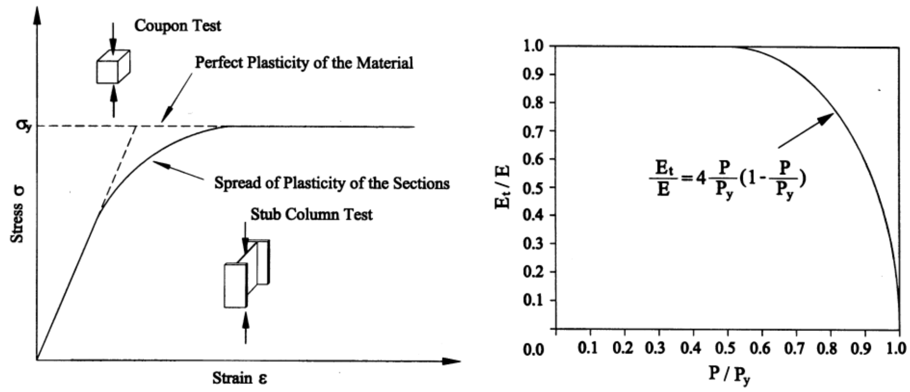
Fig. 5 Two-surface plastic hinge model by Liew et al. [205]

Hilmy and Abel [206], Al-Bermani et al. [207] and Al-Bermani and Zhu [208] used this method for the nonlinear dynamic analysis of 2D inelastic frames [206] and 3D frames [207, 208]. Liew et al. [205] also employed the concept of bounding surface to develop a nonlinear inelastic model for the assessment of 2D steel frames in fire as shown in Fig. 5. In this model, the plastic-hinges allow to form at the element ends and mid-length section. If the plastic-hinge is formed at the mid-span section, the element is subdivided into two new elements (see Fig. 5b) and the inelastic stiffness of the initial element is obtained from the inelastic stiffness of two sub-elements using the static condensation method. The application of this element technique was extended by Liew and Tang for 3D tubular frames [209] and by Chen et al. [210] for buildings with thin-walled cores. El-Tawil and Deierlein [211, 212] used two versions of the bounding surface model to develop a force-based element for 3D frames composed of steel, RC and steel-concrete composite members. This model was extended by Jin and El-Tawil [213] to simulate the inelastic cyclic behaviour of steel brace members.

Recently, Yang et al. [214] extended the rigid-body-based procedure to include the plastic-hinge model for the elasto-plastic analysis of planar frames.

4.2.2.2. Tangent modulus plastic-hinge model

In this model, the elastic modulus of a member is reduced to describe the gradual stiffness degradation due to residual stresses. White [215] adopted the column research council (CRC) tangent modulus to model the gradual yielding along the member due to residual stresses (see Fig. 6). It was concluded that this method performs well for the member under strong-axis bending, but it is insufficiently accurate for the member under weak-axis bending. To overcome this shortcoming, Ziemian and McGuire [216] modified the CRC tangent modulus based on calibrating the plastic-hinge analysis to the plastic-zone analysis for a member under axial force and weak-axis bending moment. The results indicated that the proposed tangent modulus can provide accurate results for both strong- and weak-axis behaviours. To account for the effects of different residual stress patterns for different wide flange cross-sections, Zubydan [217] proposed an empirical equation for the tangent modulus of steel members based on numerical results obtained from the plastic-zone analysis. This work was later extended by author for steel members under axial force and weak-axis bending moment [218] and under axial force and biaxial bending moments [219]. Kim et al. [220] also proposed an empirical equation for the tangent modulus of I-shaped cross-section members under different residual stress patterns, but it was based on the finite element modelling of axial members by shell elements.



(a) Inelasticity due to residual stresses (b) CRC tangent modulus E_t

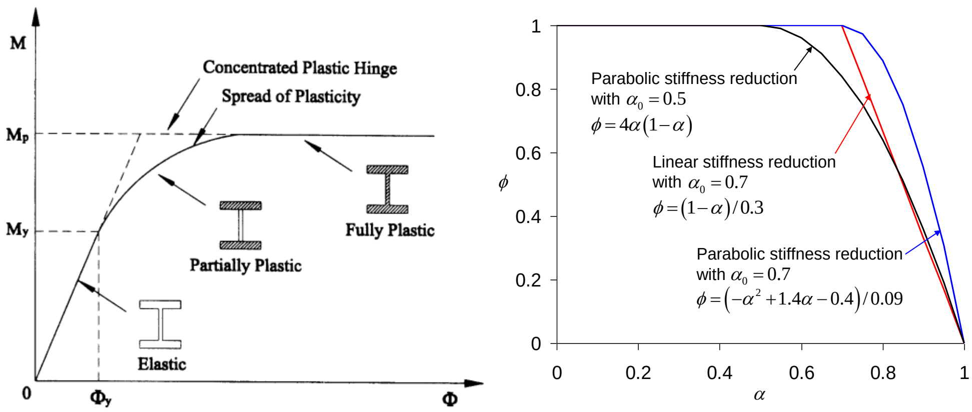
Fig. 6 Modified tangent modulus model

4.2.2.3. Refined plastic-hinge model

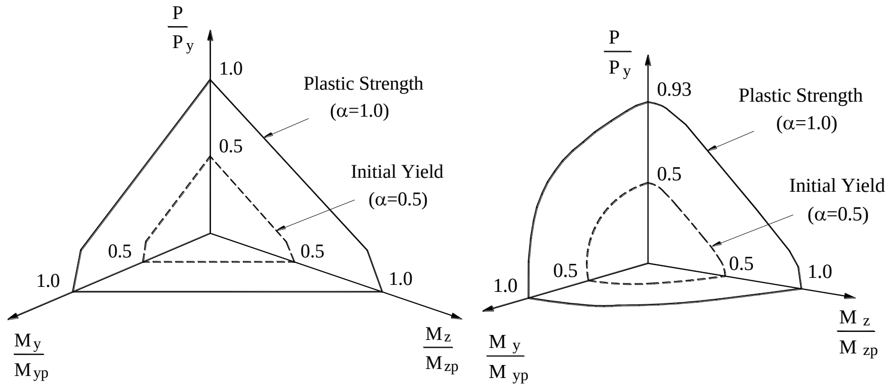
Al-Mashary and Chen [221] proposed a simplified plastic-hinge method in which the gradual yielding over the cross-sections was simulated by pseudo rotational springs attached to the two element ends as shown in Fig. 7. The gradual degradation of the spring stiffness begins when a member reaches the initial yield surface which is taken as a certain percentage of the full yield surface. When the member reaches the full yield surface, the plastic-hinge is formed, and the stiffness is reduced to zero. The spring stiffness is superimposed in the beam-column stiffness matrix using static condensation method. The P - δ effect was also included in the analysis using stability functions. King et al. [222] modified the plastic-hinge method to account for the gradual yielding at the cross-section of members. The gradual stiffness degradation from an initial yield state (when the first yielding occurs in the extreme fibres of the cross-section) to a fully inelastic state (when a plastic-hinge forms in the member) was represented by a plasticity factor varying linearly from zero (at the yield moment) to 1.0 (at the plastic moment). The effect of residual stresses is accounted for using an initial yield surface instead of the tangent modulus concept, but the P - δ effect was neglected. This method provides noticeable improvements over the elastic plastic-hinge approach. King and Chen [223] extended this method to a member under weak-axis bending. It was found that the stiffness of the member under weak-axis bending degrades more significantly than that of the member under strong-axis bending, and thus it should be considered in the inelastic analysis. King [224] developed a refined plastic-hinge model which accounted for partial plasticity at its two ends. The gradual stiffness degradation from an elastic state to the full plastic state is dependent on the slope of the bending moment-curvature relationships. A refined plastic-hinge model with plastic springs at the two ends was used for the nonlinear inelastic analysis of semi-rigid steel frames by Yau and Chan [225], steel frames by Chan and Chiu [226], composite beams by Iu [227] and composite frames by Iu et al. [228] and Liu et al. [229]. Liu et al. [230, 231] added an additional spring somehow between the two element ends to allow the formation of additional plastic-hinge.

Fig. 7 Refined plastic hinge model with plastic springs

Liew et al. [232, 233] proposed a refined plastic-hinge method for advanced analysis/design of steel frames as shown in Fig. 8. In this method, the gradual yielding due to residual stresses and axial force was considered using tangent modulus concept (see Fig. 6), whilst the gradual yielding associated with bending moments (see Fig. 8a) was accounted for using a stiffness degradation function (see Fig. 8b) expressed in terms of stress resultants based on the yield surfaces (see Fig. 8c,d). If the force point moves somehow between the initial and full yield surfaces, the stiffness of the element is proportionally reduced by the stiffness degradation function to simulate the gradual yielding. When the force point moves beyond the full yield surface, the plastic-hinge is formed. The P - δ effect was accurately captured using the stability functions. Liew et al. [234, 235] extended their work in [232, 233] to semi-rigid steel frames. Landesmann and de Miranda Batista [236] also applied the refined plastic method to semi-rigid steel frames, but the tangent modulus was modified according to Brazilian standard and Eurocode 3. Kim and Chen [237-239] presented three practical design methods for 2D steel frames: (1) explicit imperfection modelling method; (2) equivalent notational load method; and (3) further reduced tangent modulus method. Kim and Chen [240] further applied their proposed design methods to steel frames under weak-axis bending.



(a) Inelasticity due to flexure (b) Stiffness reduction models



(c) AISC-LRFD yield surface (d) Orbison yield surface

Fig. 8 Refined plastic hinge model

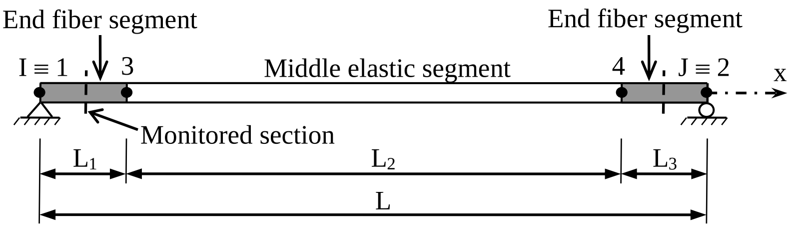
It should be noted that the refined plastic-hinge method proposed by Liew et al. [232, 233] ignores the local buckling and lateral-torsional buckling effects. Extensive research efforts therefore have been devoted to improving it. For example, Kim et al. [241] accounted for strain reversal effect, Avery and Mahendran [242] and Kim and Lee [243] accounted for the local buckling of 2D steel frames comprised of non-compact cross-sections. In their study, the local buckling was implicitly included for using the practical design equations from AISC-LRFD or AS4100 Specifications. Kim and Lee [244] incorporated the practical LRFD equation in the refined plastic-hinge method to account for the lateral-torsional buckling effect. To model distributed loads, Kim et al. [245] employed the moving node strategy to detect a plastic-hinge forming within the element under distributed loads.

The extension of the refined plastic-hinge method from 2D frames to 3D frames was carried out by Kim et al. [246]. The 3D model in [246] was also extensively improved by Kim and his colleagues to account for the semi-rigid connections and shear deformation effect [247-249], local buckling effect [250], lateral-torsional buckling effect using practical LRFD equations [251] and virtual work equation [252], distributed loads [253] and earthquake loads [254]. Kim and his colleagues extended this method for the performance based design of steel arch bridges [255] and for the optimal design of steel frames [256, 257].

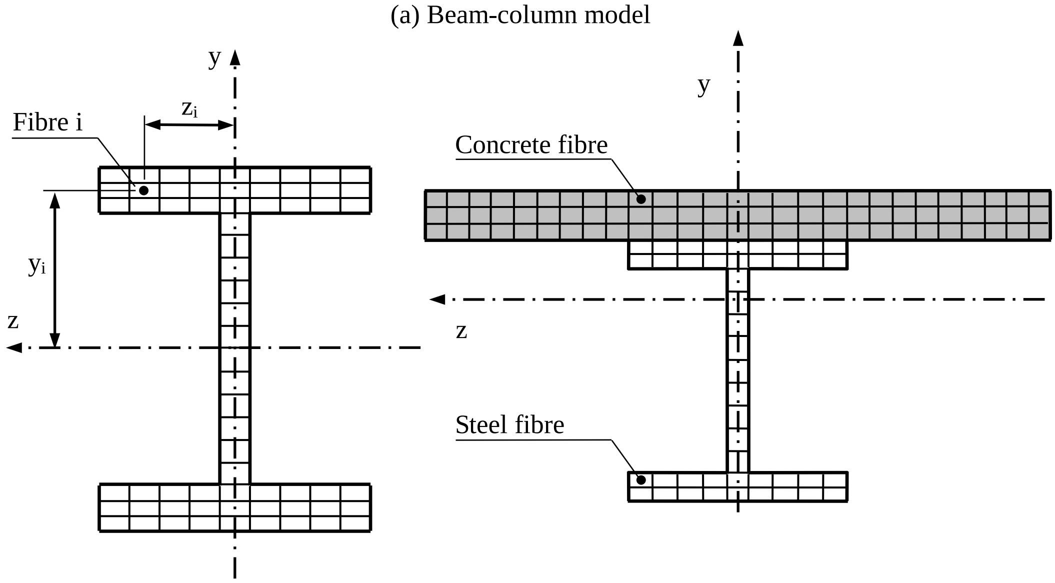
4.2.2.4. Fibre plastic-hinge model

The fibre plastic-hinge method was developed to overcome the shortcoming of the refined plastic-hinge model by eliminating the use of the stress resultant yield surfaces which are not always available for the cross-sections. In this model, the cross-sections at the two ends are partitioned into many fibres and the inelastic behaviour of materials is captured by tracking the stress-strain relationship of each fibre. The effect of residual stresses is therefore explicitly considered. Powell and Campbell [258] developed a 3D fibre plastic-hinge element and implemented in the computer program DRAIN-3DX (Type 08) for the nonlinear analysis of 3D inelastic frames subjected to static and dynamic loading. The stiffness matrix of the element was obtained by inverting the flexibility matrix of the elastic beam and two zero-length plastic-hinges. The element accounted for the P - Δ effect, but the P - δ effect was neglected. The element was later improved by Kim et al. [259] to account for the P - δ effect through the use of stability functions. Instead of using the plastic-hinge with a yield surface, Ngo-Huu et al. [260] used the fibre plastic-hinge concept in the refined plastic-hinge element for the nonlinear analysis of 3D inelastic steel frames. Krishnan and Hall [261] also presented a fibre plastic-hinge element composed of an interior elastic segment and two exterior nonzero-length fibre hinge segments at the two ends. The length of the fibre segment is obtained from the calibration with plastic-zone method. Krishnan [262] extended his work in [261] by adding two elastic segments between nonlinear segments. This element has five segments including three nonlinear fibre sections and two elastic segments and later is also implemented in the computer program FRAME3D. Ngo-Huu and Kim [263] used the same fibre plastic-hinge concept of Krishnan and Hall

[261] to develop a fibre plastic-hinge element for the nonlinear inelastic analysis of 3D steel frames as shown in Fig. 9. The P - δ effect which was neglected in [261] was accurately captured using the stability functions. By calibrating with the plastic-zone analysis using shell element of ABAQUS, it was shown that the length of the fibre segment has negligibly small effect on the predicted strength of frames. Ngo-Huu and Kim [264] extended their work in [263] to steel-concrete composite frames.



(a) Beam-column model



(b) Partition of steel sections (c) Partition of composite sections

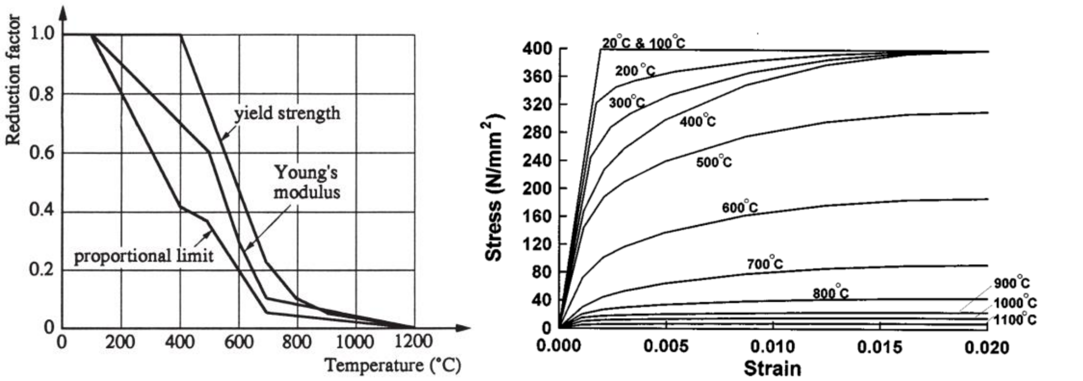
Fig. 9 Fibre plastic hinge model composed of three segments [263]

The shortcoming of the fibre plastic-hinge element with fixed plastic end zone length is that it is unable to capture the inelastic behaviour along the length of an element under distributed loading. To overcome this limitation, Lee and Filippou [265] proposed a new fibre plastic end zone element with variable lengths of plastic end zones. This element was based on the force formulation with only one integration point at each plastic end zone, but it accounted for the spread of plasticity along the length of plastic end zones. Mazza [266] developed a three-zone element for the inelastic analysis of steel and RC frames under earthquake loading. This element was divided into three zones including an elastic central zone and two plastic end zones with different lengths. The length of each plastic end zone is initially zero and increases by the increase of loading. The inelastic behaviour of the plastic end zones was modelled by the stress resultant interaction surface.

5. Analysis of frames in fire and explosion

Extensive research work has been carried out to develop theoretical program for investigating the behaviour of steel and steel-concrete composite frames in fire and explosion. Burgess et al. [267] proposed a secant stiffness method for the analysis of steel beams under fire conditions using moment-curvature-temperature relationships. This method was extended by Burgess et al. [268] to include non-uniform temperature distribution across the cross-section. Burgess et al. [269] used this method to study the behaviour of beams in fire under the variation of beam spans, cross-sectional sizes and boundary conditions. The development of a computer program VULCAN by Structural Fire Engineering Research group at the University of Sheffield is based on modifying the INSTAF program [153]. Saab and Nethercot [270] included temperature effect in INSTAF program, which was used to simulate the behaviour of 2D steel frames under fire conditions. This work was extended by Najjar and Burgess [271] to 3D steel frames and by Bailey [272] to include semi-rigid connections, lateral-torsional buckling effect and shell elements for modelling floor slab at elevated temperature. Sun et al. [273] further extended the capability of the VULCAN program to predict the complete response of steel and composite buildings. The procedure combines alternate static and dynamic analyses. Static analysis is used to trace the behaviour of the structure at changing temperature until buckling happens, and then an explicit analysis is used to simulate the response of the structure until stability is regained. This procedure was employed by Sun et al. [274] to examine the progressive collapse of steel frames with different bracing systems under different fire conditions.

Liew et al. [205] used a two-surface plastic-hinge model to assess the performance of 2D steel frames in fire. The model was extended by Liew et al. [275] and Ma and Liew [276] to predict the response of 3D steel frames in fire. Liew and Chen [277] presented a procedure for the nonlinear analysis of inelastic steel frames under localized explosion and fire. In this procedure, the plastic-zone method was used to model structural members under direct action of an explosion, whilst the structural members which are away from the influence zone of the explosion were modelled using the two-surface plastic-hinge element to save computational time. This approach is therefore computationally efficient for modelling large-scale structures under localized explosion and fire. The effect of elevated temperature on the material properties of steels was taken into consideration based on Eurocode 3 as shown in Fig. 10. Chen and Liew [278] used the mixed element concept for predicting the response of steel frames under fire and explosion. The B31 beam element and S4R shell element available in ABAQUS were used, and the connection between them was modelled by means of a kinematic coupling. However, the structural members under the direct action of explosion were modelled using shell elements instead of fibre beam-column elements as in the case of Liew and Chen [277].



(a) Reduction factors (b) Stress-strain relationship of S400 steel

Fig. 10 Typical material properties of steel at elevated temperatures according to Eurocode 3

Song et al. [279] proposed an adaptive nonlinear analysis of steel frames under fire and explosion. The adaptive method starts from the nonlinear analysis of elastic frames with a minimal mesh, typically using one quartic element per member. During the elastic analysis, the element state will be checked in predefined zones. If the element exceeds the elastic limit, it is replaced by an appropriate mesh composed of the elastic element and inelastic fibre element. This mesh refinement process significantly saves computational time. The efficiency of the adaptive analysis was verified by Izzuddin [280] for 2D steel frames under fire and explosion.

Iu and Chan [281] used a refined plastic-hinge model with plastic springs at the two ends for the nonlinear analysis of 2D inelastic steel frames under fire. This work was improved by Iu et al. [282, 283] to account for the effect of the cooling behaviour of structures during the cooling phase [282] and the effect of catenary action due to the axial force [283]. This element was also employed by Iu [284] for the nonlinear fire analysis of 3D steel frames by using based on an equivalent thermal load procedure. Toh et al. [285] presented a nonlinear inelastic model for 2D steel frames at elevated temperature. The inelastic behaviour was considered using the elastic plastic-hinge method, whilst the geometric nonlinearity was included using the CR formulation and stability functions. Landesmann et al. [286] modified the refined plastic-hinge element to include temperature effects. Both tangent modulus and stiffness degradation model were modified. Landesmann [287] extended the previous work in [286] to semi-rigid steel-concrete composite frames under fire. Junior and Creus [288] presented a simplified analysis of inelastic steel frames in fire by modifying the tangent modulus of members. Recently, Prakash and Srivastava [289] developed a distributed plasticity element for advanced analysis of 3D semi-rigid steel frames under fire. This element was formulated in the framework of direct stiffness method. Numerical results indicated that it is suitable for various applications including performance-based fire design, progressive collapse analysis, uncertainty analysis and optimization of steel structures under both standard and realistic fire scenarios [289]. Barros et al. [290] proposed the use of the refined plastic hinge model coupled with the strain compatibility method to simulate the nonlinear behaviour of steel frames under fire. The use of the strain compatibility method allows more realistic analysis since the stress-strain relationship is taken from the specifications of structural design codes.

6. Analysis of steel-concrete composite structures

6.1. Composite beams with partial interaction

6.1.1. Analytical model

One of the earliest analytical models for the linear elastic analysis of composite beams with partial interaction was introduced by Newmark et al. [291]. In this model, both steel beam and concrete slab were separately modelled using two EB beams, whilst the slip between them was modelled by means of a linear interface. Based on the Newmark's model, many researchers have developed 'exact' stiffness matrices for the linear elastic analysis of composite beams by solving the differential governing equations. For example, Faella et al. [292] derived the exact analytical expressions of nodal force vector and stiffness matrix of a six DOFs element for composite beams. Faella et al. [293] improved their previous work in [292] to include the nonlinear interface slip and the effects of concrete slab cracking and tension stiffening. Faella et al. [294] proposed an element allowing for direct determination of the exact analytical expressions of the force vector and stiffness matrix instead of inversing the flexibility matrix as in the case of their previous work in [292]. Ranzi et al. [295] derived an exact stiffness matrix of a six DOFs element for composite beams using the direct stiffness method. Ranzi and Bradford [296] extended their previous work in [295] to derive the stiffness matrix of an eight DOFs element. A comparative study of four different methods including the analytical method, direct stiffness method, finite difference method and FEM was also carried out by Ranzi et al. [297]. Gara et al. [298] and Ranzi et al. [299] developed analytical models for composite beams considering both transverse and longitudinal partial interactions.

The development of analytical models accounting for the shear deformation effect in composite beams with partial interaction has attracted many researchers. Ranzi and Zona [300] developed both analytical and FEM models for composite beams. In these models, the steel beam and concrete slab were separately modelled by Timoshenko and EB beam theories. Xu and Wu [301] adopted Timoshenko beam theory to develop an analytical model for composite beams accounting for the shear deformation effect, rotary inertia and axial force. Based on the kinematic model of Xu and Wu [301], Martinelli et al. [302] derived the exact analytical expressions of the nodal force vector and stiffness matrix of a six DOFs element for shear deformable composite beams. A comparison of analytical models for shear deformable composite beams was presented by Martinelli et al. [303].

6.1.2. Displacement-based element

Yasunori et al. [304] presented a fibre element with 12 DOFs for the inelastic analysis of composite beams by assuring continuity of rotation and mean axial strain in the two components. The element proposed by Daniels and Crisinel [305, 306] has 10 DOFs in which 2 DOFs adding through the internal nodes to account for cubic variation of the interface slip. Oven et al. [307] presented a fibre element for the nonlinear inelastic analysis of steel-concrete composite beams by modifying the INSTAF program [153]. The nonlinear analysis of inelastic composite beams carried out by Iu [227] was based on a UL cubic element and a refined plastic-hinge model with two plastic springs at the two ends. The partial composite action between concrete slab and steel beam was implicitly accounted for using the effective moment of inertia of composite sections. Fong and Chan [308] presented an advanced analysis method for composite beams including geometric and material nonlinearities as well as geometric imperfections. The geometric nonlinearity was accurately captured using only a single element proposed by Chan and Zhou [122], whilst the inelastic behaviour was accounted for using a refined plastic-hinge model with the initial and full yield surfaces of composite sections.

Dall'Asta and Zona [309] presented a fibre element with 16 DOFs for the inelastic analysis of steel-concrete composite beams (see Fig. 11). The accuracy of the element was significantly improved by using the higher-order polynomials. The axial displacements were approximated using the fourth-order polynomials by adding three internal nodes, whilst the deflections and rotations were approximated using fifth-order polynomials by adding an internal node. The inelastic behaviour was accounted for using a fibre model in which the cross-section is divided into rectangular strips parallel to the longitudinal direction. Numerical integration with the trapezoidal rule through the thickness and the Gauss-Lobatto rule along the length was adopted. The results indicated that the proposed higher-order element provides accurate results, but it is more computationally expensive due to requiring a static condensation procedure. Battini et al. [310] proposed 8 DOFs element for the large deflection analysis of two-layer composite beams. The exact stiffness matrix derived from the exact solution of differential governing equations was used to avoid the locking problem. The geometric nonlinearity was included using CR approach. This model was extended by Hjiaj et al. [311] to include the shear deformation effect by using Timoshenko beam theory.

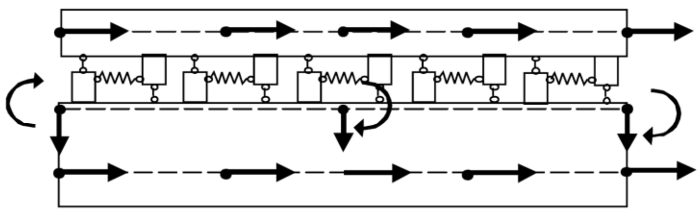
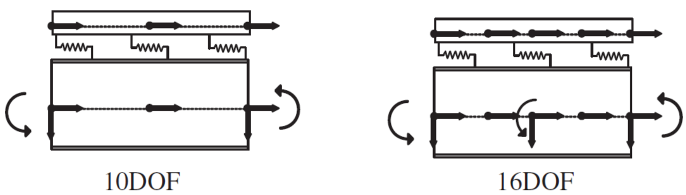
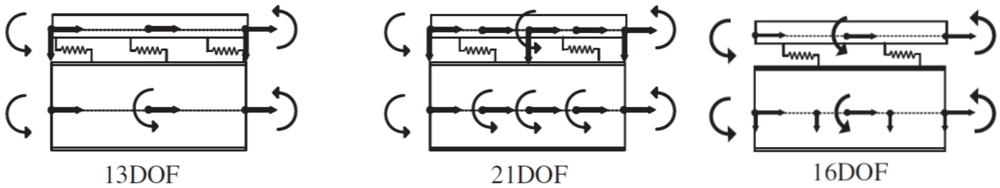


Fig. 11 Displacement-based Newmark models for composite beams [309]

A fibre element including both geometric and material nonlinearities was developed by Sousa Jr et al. [312] and Cas et al. [313] for composite beams. Ranzi et al. [314] presented a nonlinear element for composite beams considering both transverse and longitudinal partial interaction. Zona and Ranzi [315] presented a comparison of three different models for composite beams. They are: (1) the Newmark's model using EB beam theory for both concrete slab and steel beam, (2) the EB-T model using EB beam theory for the concrete slab and Timoshenko beam theory for the steel beam and (3) the T-T model using Timoshenko beam theory for both steel beam and concrete slab. Three corresponding nonlinear elements adopted in the analysis are: (1) 10 DOFs and 16 DOFs element [309], (2) 13 DOFs and 21 DOFs elements [300] and (3) the proposed 16 DOFs element as shown in Fig. 12. Verification results indicated that the Newmark's model only gives a good prediction for the composite beam under bending, but it significantly overestimates the strength of the composite beam under shear action due to ignoring the shear deformation effect.



(a) Newmark kinematic models



(b) EB-Timoshenko kinematic models (c) Timoshenko-Timoshenko kinematic model

Fig. 12 Displacement-based models for composite beams [315]

6.1.3. Force-based element

Salari at al. [316] developed a force-based fibre element for the inelastic analysis of composite beams as shown in Fig. 13. The element accounts for a distributed shear interaction through the use of a cubic polynomial, but it encounters convergence problems when the bond-slip law softens and fails. This limitation was overcome in a special force recovery procedure developed by Salari and Spacone [317]. The force-based fibre element proposed by Ayoub [318] was similar to the one developed by Salari at al. [316]. However, the partial interaction between concrete slab and the steel beam was modelled by a simplified approach through the use of a spring element with a linear variation of the bonding stress along the element length. Valipour and Bradford [319] extended the force-based fibre element in [175] to steel-concrete composite beams. The slip along the length was calculated using the Simpson integration scheme with a parabolic interpolation. The element was proved to be numerical stability since it was based on the total secant stiffness approach. Based on the CR approach, Nguyen et al. [320] developed a force-based element for shear deformable two-layer beams considering both geometric and material nonlinear effects. The force-based formulation was also employed by Monti and Spacone [321] to develop a new element for the inelastic analysis of RC beams under cyclic loads. This element is a combination of the bar element with bond slip by Monti et al. [322] and the force-based element by Spacone et al. [172], and explicitly accounts for the bond slip between surrounding concrete and reinforcing bars.

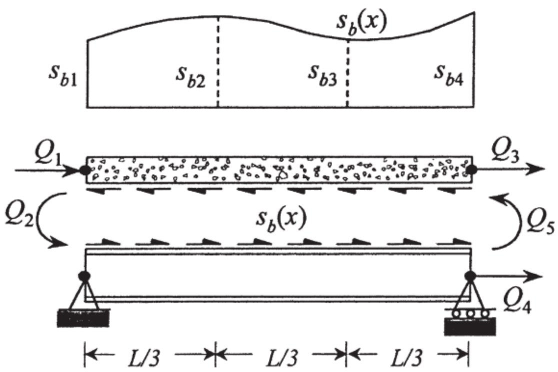
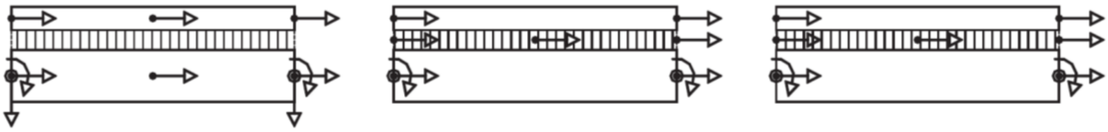


Fig. 13 Force-based elements with cubic bond stress distribution for composite beams [316]

6.1.4. Mixed element

To overcome the limitations of the displacement-based element and the difficulty of the force-based element, Ayoub [323] and Ayoub and Filippou [324, 325] developed a mixed element, which was formulated using independent interpolation functions for transverse displacement and internal forces. This element was extended by Ayoub [326] to the inelastic analysis of RC beams with bond-slip under cyclic loads. Ayoub [327] also evaluated the performances of mixed elements and displacement-based elements in predicting the behaviour of various composite beams and columns. The comparison results confirmed the superiority, practicality and advantages of the mixed element over displacement-based one. Limkatanyu and Spacone [328, 329] presented a theoretical framework of mixed, displacement-based and force-based formulations of beam elements accounting for the bond-slip in the reinforcing bars. Sun and Bursi [330] analysed steel-concrete composite beams with shear lag using three different elements: the displacement-based model, the force-based model [316] and the two-field mixed model [324]. Nguyen et al. [331] proposed a two-field mixed element for the inelastic analysis of composite beams. Dall'Asta and Zona [332] proposed a mixed element for the nonlinear analysis of inelastic composite beams based on a three-field mixed formulation with independent interpolation functions for the displacement, strain and stress fields as shown in Fig. 14. This element was compared and validated with displacement-based elements by Dall'Asta and Zona [333]. Lee and Filippou [334] proposed a mixed element for the inelastic analysis of composite and RC members. The element is based on three-field formulation with the relative slip considered as an independent variable. Recently, Lee and Filippou [335, 336] proposed three alternatives for the mixed formulation: mixed-displacement, mixed-mixed and mixed-force. The merits and shortcomings of the alternatives were also discussed in details. The mixed element can be used for simulating the inelastic response of RC, composite and CFST members.



(a) Displacement field (b) Strain field (c) Stress field

Fig. 14 Three-field mixed element HW112 for composite beams [332]

6.2. CFST columns

Hajjar and Gourley [337, 338] developed a displacement-based element for the nonlinear analysis of 3D CFST inelastic columns and frames under static and cyclic loadings. The geometric nonlinearity was included in a UL formulation, whilst the material nonlinearity was captured using two-surface plastic-hinge model. Hajjar et al. [339, 340] developed a 3D displacement-based fibre element for the nonlinear analysis of 3D inelastic CFST columns and frames under static and cyclic loadings. This element has 18 DOFs including 12 DOFs in a conventional fibre element and additional 6 DOFs used to account for the interlayer slip between the steel tube and in-filled concrete. Tort and Hajjar [341, 342] also developed a 18 DOFs fibre element for the nonlinear analysis of inelastic CFST columns and frames under static and dynamic loading, but it was based on a two-field mixed formulation. Separate translational DOFs were used for the steel tube and in-filled concrete as shown in Fig. 15.

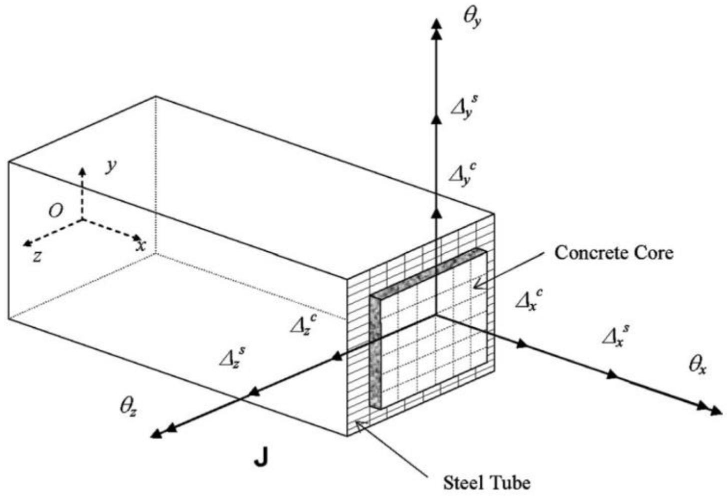


Fig. 15 Mixed element with 18 DOFs for CFST columns [342]

Aval et al. [343] proposed a displacement-based fibre element for the large deflection inelastic cyclic analysis of CFST columns which accounts for the interlayer slip between in-filled concrete and the steel tube. The 2D three-node element has 13 DOFs including 5 DOFs per end node and 3 DOFs on the middle node as shown in Fig. 16. By using two separate rotational DOFs for the steel tube and in-filled concrete, this element is capable of modelling the flexural slip which was ignored in the 18 DOFs element proposed in [339, 340]. Fong et al. [344] presented an advanced analysis method for CFST columns with initial geometric imperfection. In this method, the geometric nonlinearity was accurately captured by using only a single element proposed by Chan and Zhou [122], whilst the inelastic behaviour was captured by a refined plastic-hinge model with inelastic springs at the two ends of the member. Valipour and Foster [345] extended the application of their force-based fibre element in [176] to CFST columns under both static and cyclic loadings. The local buckling of steel tubes was also taken into account by adjusting the yield stress according to the slenderness of the plate. Thai et al. [346] presented a force-based fibre element which accounts for local buckling of the steel tubes in the CFST columns. A new stress-strain model accounting for elastic and inelastic buckling of the steel tubes was proposed and incorporated in a force-based fibre element. However, the slip between steel tube and in-filled concrete was ignored. Recently, Liu et al. [347] extended their previous work [230, 231] to CFST columns with circular and octagonal sections.

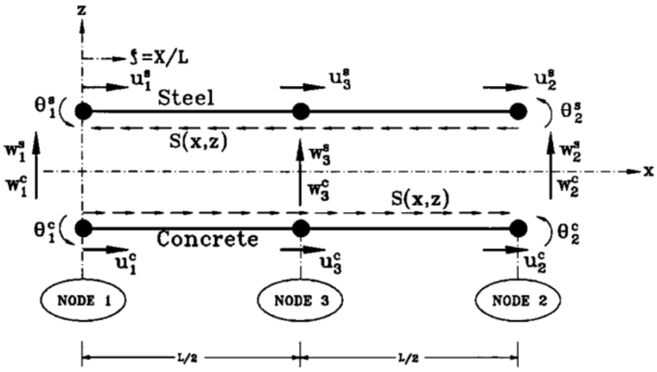


Fig. 16 Displacement-based element with 13 DOFs for CFST columns [343]

6.3. Composite frames

Fang et al. [348] proposed an efficient, robust and accurate method for predicting the ultimate load-carrying capacity of 2D concrete-steel composite semi-rigid frames. A numerical integration scheme was used to capture the gradual yielding at cross-sections and along the length. This method however was limited to composite beams with full interaction. Fang et al. [349] later proposed a simplified plastic-layer method for the inelastic analysis of composite frames with partial interaction. The layered approach is simpler and more computationally efficient than the fibre method in simulating the inelastic behaviour of materials. Liew et al. [350] proposed an analysis method for inelastic composite frames. In this method, the steel column was modelled by the plastic-hinge element, whilst the composite beam was modelled by a distributed plasticity element. The composite beam was divided into a finite number of segments. The flexural stiffness of each segment was evaluated in terms of the moment-curvature relationship of the composite cross-section which can approximately account for the partial interaction between the concrete slab and steel beam. Kim and Engelhardt [351] developed a plastic-hinge element for the inelastic analysis of composite frames under earthquake loads. A moment-rotation model was proposed and implemented in a plastic-hinge model to predict the inelastic behaviour of materials. However, the partial interaction between concrete slab and the steel beam was ignored. The nonlinear inelastic analysis of composite frames carried out by Landesmann [352] was based on the refined plastic-hinge method original developed by Liew et al. [232, 233] for steel frames. Chiorean [22] proposed a reliable and robust model for the nonlinear inelastic analysis of 3D composite frames with full interaction. The material nonlinearity was included using the force-based fibre model, whilst the geometric nonlinearity was captured using stability functions and UL formulation. This method is very computationally efficient because of requiring only one element per member. This work was recently improved by Chiorean and Buru [353] to account for partial interaction.

In addition, many works on composite frames were based on an extension from the research work on composite beams. For example, Salari and Spacone [354] extended their force-based fibre element in [316] to predict the inelastic response of partial interaction composite frames under static and cyclic loadings. Zona et al. [355] extended the displacement-based element developed by Dall'Asta and Zona [309] to simulate the inelastic seismic behaviour of composite frames. The nonlinear inelastic analysis of composite frames carried out by Iu [356] was extended from the refined plastic-hinge model developed by Iu [227] for composite beams.

7. Conclusions

The development of beam-column models for steel and composite frames has been comprehensively reviewed and discussed in this paper. A huge number of beam-column elements have been developed over the past four decades based on the displacement, force and mixed formulations. These elements have also been implemented in several computer programs for practical design and research purposes. Among the displacement-based models, the cubic element has been widely used for the nonlinear inelastic analysis of steel and composite frames because of its simplicity. However, this element is sufficiently accurate for practical use in the analysis and design of frames if P - δ effect is negligibly small. For the frames with slender members under large axial forces (significant P - δ effect), the members need to be divided into several elements in order to convert the P - δ effect into the P - Δ effect. To capture accurately P - δ effect by using the least element per member, the beam-column approach was introduced based on the use of the stability functions. Extensive work has been devoted to the development of accurate elements for the advanced analysis of steel frames by using the least element per member, and thus they can be used for the practical design of large-scale steel frames. Notable among them are PEP and stability function-based elements.

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